MODESTUM

Building the educational future: Exploring the dimensions of STEM education through 3D printing and platonic solids in university mathematics teaching

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INTRODUCTION

STEM education (Science, Technology, Engineering, and Mathematics) has emerged as an integrative approach to merge the disciplines of science, technology, engineering, and mathematics in higher education. In this context, university mathematics education represents a unique opportunity to explore the potential of 3D printing technology, with a particular emphasis on the visualization and understanding of Platonic solids. Historically, the representation of three-dimensional geometry posed significant challenges, relying mostly on two-dimensional supports and a set of manipulative materials to demonstrate fundamental properties of geometric bodies (Díez Molina & Roa González, 2017). Fortunately, current technological advancements, such as modeling software and 3D printers, offer greater versatility in presenting the world of three-dimensional geometry to students.

The convergence between STEM education and the transformative capabilities of 3D printing has generated an innovative educational horizon that redefines the learning experience in the university setting. This study delves into the intersection of these disciplines, exploring the opportunities and challenges that arise when incorporating 3D printing in the teaching of Platonic solids in higher mathematics education. Platonic solids, renowned for their unique geometric properties, offer a fertile ground to explore advanced mathematical concepts. However, simply having access to this technology does not guarantee meaningful learning experiences. It is crucial to base educational proposals on established didactic models, such as the Van Hiele model, to fully exploit the potential of 3D printing in the classroom (Díez Molina & Roa González, 2017).

3D printing technology, in turn, unleashes unprecedented educational opportunities. As Lipson and Kurman (2013) suggest, 3D printing can transform mathematical abstraction into tangible entities, providing a new dimension for understanding. Additionally, Ford and Minshall (2018) argue that the emergence of additive manufacturing and 3D printing technologies is introducing industrial skills deficits and opportunities for new teaching practices in a variety of subjects and educational environments. Ng et al. (2022) highlight the importance of 3D printing as an innovative way to visualize mathematical concepts, enabling students to develop mathematical and design thinking, as well as digital skills and mindsets. However, the effective implementation of this multidisciplinary integration requires overcoming conceptual and technological barriers, as pointed out by Mishra and Koehler (2006).

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Furthermore, it is essential to consider the legal and ethical implications of this emerging technology. According to Frandsen (2014), the economic consequences of unauthorized copying of products with personal 3D printers for intellectual property owners are expected to be significant. These challenges underscore the need for a holistic approach that integrates both pedagogical and legal and ethical aspects. Beltrán-Pellicer and Rodríguez-Jaso (2017) argue that the utilization of 3D printing in mathematics education justifies its use by facilitating a deeper understanding of mathematical concepts through tangible visualization and manipulation. Their exploratory study supports the integration of 3D printing into educational practices to enhance the learning experience in mathematics classrooms.

The introduction of 3D printing technology into university mathematics education not only offers new ways of understanding abstract concepts but also promotes a hands-on, experimental approach to learning. By allowing students to interact with physical models of Platonic solids and other mathematical objects, 3D printing provides an immersive learning experience that transcends the boundaries of traditional classrooms. This active, experiential approach not only enhances retention and understanding of the material but also fosters the development of problem-solving skills and critical thinking.

Moreover, the integration of 3D printing into university mathematics education opens up new avenues for interdisciplinary collaboration and innovative research. By providing students and faculty with access to advanced design and manufacturing tools, it creates an environment conducive to exploration and experimentation in fields such as computational geometry, scientific visualization, and reverse engineering. This convergence of disciplines not only enriches the educational experience but also drives the advancement of knowledge in interconnected areas, preparing students to tackle the complex challenges of the 21st century.

DEVELOPMENT

The implementation of 3D printing, far from being just a technological tool, becomes a bridge between mathematical theory and practical experience. In the words of Papert (1980), "Technology is the only way to give students a direct experience with abstract mathematical concepts." The ability to 3D print Platonic solids allows students to touch and manipulate these shapes, providing a tangible connection to the underlying geometric principles. Beltrán-Pellicer and Rodríguez-Jaso (2017) support this approach, emphasizing that 3D printing in mathematics education facilitates a deeper understanding of mathematical concepts through the tangible visualization and manipulation of objects. Their study highlights the potential of 3D printing to enhance mathematical comprehension by providing students with hands-on experiences that bridge abstract theory and practical application.

However, we must not lose sight of the need for careful implementation. As Mishra and Koehler (2006) caution, "Integrating technology into teaching is not simply adding electronic devices but changing the way we conceive and design learning experiences." 3D printing, therefore, should be integrated in a way that amplifies and enhances mathematical understanding, not simply as a technological embellishment. According to Díez Molina and Roa González (2017), implementing 3D printing in the classroom should follow established educational models, such as the Van Hiele model, which structures the learning process into distinct phases. This approach ensures that the introduction of 3D technology in teaching three-dimensional geometry is grounded in a theoretical framework that maximizes its educational impact.

The implementation of 3D printing in university mathematics education has been the subject of study by various researchers, such as Davy Tsz Kit Ng, Ming Fung Tsui, and Manwai Yuen. These authors highlight that 3D printing offers a unique opportunity to materialize abstract mathematical concepts, providing students with a tactile experience that goes beyond traditional twodimensional representations (Ng et al., 2022). Moreover, according to the same study by Ng et al. (2022), 3D printing facilitates the active construction of mathematical knowledge, following the principles of the constructivist approach proposed by Piaget (1968). This technology allows students to create and physically examine concrete objects based on abstract concepts, contributing to a deeper and more meaningful understanding of geometric principles.

On the other hand, studies by Ng et al. (2022) also highlight the role of 3D printing in developing digital skills among students. It has been observed that the integration of 3D printing in mathematics education promotes mastery of modeling tools and understanding of engineering processes, enhancing students' ability to visualize mathematical concepts and creatively address problems.

Additionally, as highlighted by Ford and Minshall (2018), the inclusion of 3D printing in school curricula provides opportunities for different learning styles to be practiced, including experiential learning and failure. This pedagogical perspective emphasizes the importance of hands-on experiences in enhancing student engagement and learning outcomes.

Furthermore, as Akerson (2013) reflects, the acronym "STEM" has evolved to encompass various interpretations, including "STEAM" and "STIM," reflecting the interdisciplinary nature of these fields. Despite these variations, the common goal remains the promotion of scientific literacy and education.

The Next Generation Science Standards (NGSS) emphasize the importance of Nature of Science (NOS) concepts in K-12 education (Akerson, 2013). These concepts, integrated into science and engineering practices, include understanding the methods of scientific investigations, the reliance on empirical evidence, the openness of scientific knowledge to revision, and the role of scientific models, laws, mechanisms, and theories in explaining natural phenomena.

Lastly, to illustrate how 3D printing can be effectively integrated into university mathematics education, let us consider the use of modeling software such as GeoGebra. This software provides a versatile platform for creating three-dimensional models of

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- $=$ GeoGebra

Figure 1. 2D and 3D images of the dodecahedron [\(https://www.geogebra.org/m/epzzny7m\)](https://www.geogebra.org/m/epzzny7m)

Figure 2. Tetrahedron development [\(https://www.geogebra.org/m/pvc6pqu5\)](https://www.geogebra.org/m/pvc6pqu5)

geometric solids and exploring their properties. Step by step, students can design and manipulate these models, deepening their understanding of geometry while developing digital skills and the ability to creatively address mathematical problems.

Modeling Platonic Solids with GeoGebra Follows the Following Construction Protocol

2D and 3D graphic view (see **Figure 1**):

- 1. Enter A=(0,0)
- 2. In 2D graphic view 1, construct slider r in [0,5], increment 0.1
- 3. Then slider n in [3.5], increment 1
- 4. Circle (center A, radius r). Move r to value 3 or 4 to vary radius
- 5. Construction sequence list1=sequence [(r;(360j/n)°),j,0,n] or (360(j/n)° or [(r;(360j°/n)),j,0,n]

With the sequence tool [<exp>,<exp>,<exp>], move n to verify that 3, 4, and 5 equidistant points are generated on the circumference.

- 6. Polygon [list1] is the list of points
- 7. Activate 3D graphic view and use the command: Tetrahedron[<pt>,<pt>,<pt>] and the command element [>list>,<n°(position)>] and write

Tetrahedron (see **Figure 2**) [Element[list1];1], Element[list1;2]], Element[list1,3]]

8. Idem octahedron (see **Figure 3**) [Element[list1];1], Element[list1;2]], Element[list1,3]]

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Figure 4. Icosahedron development [\(https://www.geogebra.org/m/aq66hew6\)](https://www.geogebra.org/m/aq66hew6)

- 9. Idem icosahedron (see **Figure 4**) [Element[list1];1], Element[list1;2]], Element[list1,3]]
- 10. Move n to number 4 and in 3D graphic view cube (see **Figure 5**) [Element[list1];1], Element[list1;2]], Element[list1,3]]
- 11. Move n to 5 and in 3D graphic view write dodecahedron (see **Figure 6**) [Element[list1];1], Element[list1;2]], Element[list1,3]]
- 12. Place the slider at n=3 and activate 2D graphic view 1. Checkbox and place tetrahedron and see in VA the name) and click on choose and move.

Figure 6. Profile view of a dodecahedron [\(https://www.geogebra.org/m/j95aye2v\)](https://www.geogebra.org/m/j95aye2v)

- 13. Hide elements in 2D graphic view 1 by holding right click, select the object to hide, right click on properties, select advanced option, and deactivate 2D graphic view. Hide the Cartesian plane.
- 14. Create the checkbox for icosahedron and octahedron, in each case hide the elements of 2D graphic view 1.
- 15. Move n to 4 and create the checkbox for CUBE (hexahedron).
- 16. Move n to 565 and create the checkbox for dodecahedron.
- 17. In this activated there will be more than one checkbox. Deactivate the checkboxes, each one has a name, logical value g is called g, j, h, k, i
- 18. Click on 2D graphic view 1 and go to properties, script program, and write:

J=false

H=false

K=false

I=false

Ok and accept

The Platonic solids will be depicted in the 3D Graphics View: This tool will allow a detailed three-dimensional visualization of the geometric solids, facilitating understanding of their structure and characteristics. With the ability to activate only one solid at a time using the created control checkboxes, users can focus on each figure individually, promoting deeper and more meaningful exploration.

Using the created control checkboxes, a single solid can be activated and the "development" tool employed: The control checkboxes provide precise control over which Platonic solid is displayed in the 3D Graphics View, enabling a more focused and personalized learning experience. Additionally, the "development" tool provides an interactive way to explore the features and

The following figure shows the 2D Graphics View and the 3D Graphics View, both with the development of a Platonic solid: This dual representation in 2D and 3D offers a comprehensive perspective of the solid, allowing users to observe both its shape in two dimensions and its three-dimensional structure. This facilitates the connection between the two-dimensional representation and the actual figure, promoting a deeper understanding of the solid's geometry.

This creation is available at: By providing direct links to the creations in GeoGebra, access to the Platonic solids models is made easier for users. This encourages collaboration and resource sharing among students and educators, as well as independent exploration outside the traditional learning environment. Users can access the models anytime and anywhere, fostering continuity of learning and self-directed practice.

Next, all the Platonic solids modeled with GeoGebra are displayed in the figures. Each solid is represented in both the 2D Graphics View and the 3D Graphics View, providing a comprehensive understanding of its structure and geometry. Direct links to each creation in GeoGebra facilitate access to the individual models, allowing users to explore and study each solid in detail and at their own pace.

Printing a File from GeoGebra

To print a file from GeoGebra, it is crucial to understand the format requirements of the 3D printer that will be used. Since different printers may support various file types such as STL, GCOD, among others, it is essential to know the specifications of the printer being used. In this context, a printer that only accepts GCOD files has been chosen, necessitating the conversion of files generated in GeoGebra to this specific format (see **Figures 7** and **8**).

Figure 7. Where to find the download tool [\(https://www.geogebra.org/classic/ybjbfqgj\)](https://www.geogebra.org/classic/ybjbfqgj)

Figure 8. How to download the STL file in GeoGebra [\(https://www.geogebra.org/classic/ybjbfqgj\)](https://www.geogebra.org/classic/ybjbfqgj)

Once the Platonic solids have been created in GeoGebra, they can be exported as STL files, marking a crucial step in the preparation process for 3D printing. This export action is performed to generate STL files from the original .ggb files. In this section, we will detail the process of generating STL files for the five Platonic solids using GeoGebra. Subsequently, the free software Repetier Host, which can be downloaded at no cost, will be used to convert the STL files into G-COD files, required for printing on the selected 3D printer (see **Figures 9** and **10**).

Figure 9. Graphical view (Source: Authors' own elaboration, using Reperier-Host V2.1.6)

Figure 10. View of the tetrahedron file (Source: Authors' own elaboration, using Reperier-Host V2.1.6)

Now, we will focus on explaining step by step how to generate the STL file of the Tetrahedron. Once this STL file has been successfully saved, we will proceed to open it using the Repetier Host software, as demonstrated in the following section.

As a fun activity for students, once the Platonic solids have been printed, they can be creatively utilized to decorate a Christmas tree. This activity not only fosters creativity and artistic expression but also reinforces the understanding of geometric shapes and their properties in a festive context. Students can explore various design options and arrangements using the printed solids, enhancing their spatial reasoning and visualization skills while enjoying the holiday spirit.

Furthermore, decorating a Christmas tree (see **Figure 11**) with 3D-printed Platonic solids (see **Figure 12**) offers an interdisciplinary learning opportunity by combining elements of mathematics, art, and holiday traditions. Students can collaborate on designing and arranging the solids, incorporating mathematical concepts such as symmetry and proportion into their festive decorations. This hands-on approach to learning promotes engagement and active participation, making the exploration of geometry and mathematical principles more enjoyable and relevant to students' lives.

Figure 11. A Christmas tree decorated with 3D-printed objects, including the five Platonic solids and additional solids. At the top of the tree is a 3D augmented reality GeoGebra logo (Source: Authors' own elaboration, using GeoGebra logo)

Figure 12. The five Platonic solids printed in 3D (Source: Authors' own elaboration, using GeoGebra Ambassador 2021/22)

Moreover, the activity of decorating a Christmas tree with 3D-printed Platonic solids encourages teamwork and collaboration among students as they work together to create a visually appealing display. By sharing ideas, problem-solving, and cooperating on the design and placement of the solids, students develop important social and communication skills while also reinforcing their understanding of geometric concepts. Overall, this festive application of 3D-printed Platonic solids adds an interactive and enjoyable dimension to learning mathematics and promotes creativity and teamwork among students during the holiday season.

DISCUSSIONS

The integration of 3D printing in university mathematics education, particularly in teaching Platonic solids, raises significant issues regarding accessibility and widespread implementation. It is essential to address logistical considerations, such as resource availability and teacher training, to ensure that this technology benefits all students.

The integration of 3D printing in teaching Platonic solids has taken mathematics education beyond the limitations of paper and chalkboard. As Lipson and Kurman (2013) emphasize, "3D printing can transform mathematical abstraction into tangible entities." This tangible dimension allows students to explore geometric shapes from physical perspectives, enhancing their understanding and appreciation of geometry. The tactile experience adds an additional layer to the assimilation of mathematical concepts, making teaching more accessible and meaningful. However, this technology poses challenges related to intellectual property and liability for damages, as Cortés (2019) mentions, which require careful management by educational institutions to avoid infringements and mitigate risks.

Despite the opportunities, we cannot ignore the inherent challenges of integrating 3D printing into STEM education. Mishra and Koehler (2006) warn that "integrating technology into teaching is not simply adding electronic devices but changing how we conceive and design learning experiences." Educators must overcome technological barriers while ensuring that technology enhances teaching and is not a distraction. Additionally, adequate training is essential for educators to effectively use 3D printing as a pedagogical tool.

The incorporation of 3D printing in teaching Platonic solids opens new pathways for the development of abstract thinking. Euclid (3rd century BCE) asserted that "there is no other form that can represent the same balance and stability as Platonic solids." By translating these abstract concepts into tangible objects, students' abstract thinking is stimulated. Visualization and manipulation of Platonic solids foster mathematical conceptualization, allowing students to explore and understand abstract principles in a more accessible format.

The convergence of STEM, mathematics, and art through 3D printing prepares students for an interconnected and technologically advanced world. The ability to conceptualize, design, and materialize mathematical concepts not only broadens academic perspectives but also equips students with applicable skills in various disciplines and professional sectors.

The integration of 3D printing into educational settings raises significant ethical and legal considerations, particularly concerning intellectual property and liability issues. The capability of 3D printing to replicate objects easily introduces challenges regarding the infringement of intellectual and industrial property rights, especially when utilizing copyrighted designs or models in educational contexts. Abolghasem (2021) highlights the importance of ethical and transparent practices in 3D printing to navigate these challenges effectively. It is imperative for educational institutions to ensure compliance with intellectual property laws and obtain appropriate licenses where necessary. Moreover, the responsibility for potential damages caused by studentdesigned and printed objects poses another critical concern. Institutions must establish comprehensive policies that outline clear guidelines for supervision and risk management to mitigate liability risks and ensure the safety of all involved parties.

Furthermore, the equitable access to 3D printing technology is crucial to prevent exacerbating existing educational disparities. The unequal distribution of access could widen the gap between students from different socio-economic backgrounds, limiting opportunities for some while advantaging others. Educational institutions must prioritize initiatives that promote equitable access to 3D printing resources, thereby fostering a more inclusive learning environment. Additionally, the use of online platforms and software for designing and printing 3D objects necessitates robust measures to safeguard students' privacy and data security. Institutions should implement stringent protocols and policies to protect sensitive information and uphold student confidentiality. Innovations such as the integration of blockchain technology, as seen in projects like ImpreAndes3D, offer promising solutions to enhance transparency and trust in the educational use of 3D printing technologies. By addressing these ethical and legal considerations thoughtfully, educational institutions can effectively harness the educational benefits of 3D printing while safeguarding against potential risks, ensuring a responsible and equitable implementation across diverse learning environments.

LIMITATIONS AND FUTURE WORK

Despite the discussed advances and benefits, this study presents several limitations that may affect the generalizability of the findings. One major limitation is the sample used, which was confined to a single university educational context. Broadening the research to include diverse educational institutions and educational levels could provide a more comprehensive understanding of the impacts and challenges of integrating 3D printing in teaching Platonic solids in mathematics.

Another significant limitation is the duration of the study, which focused on a specific implementation period. Future research could adopt a longitudinal approach to assess the long-term impact of 3D printing technology on academic performance and student motivation in mathematics. Additionally, it would be beneficial to further explore how individual differences among students, such as their level of prior technological competence, may influence educational outcomes achieved through the use of 3D printing.

In terms of future work, investigating specific pedagogical strategies that optimize the use of 3D printing for teaching complex mathematical concepts is recommended. This could include developing new instructional models based on contemporary cognitive or educational theories, such as the constructivist approach or the Van Hiele model. Furthermore, exploring how the integration of other emerging technologies, such as augmented reality or artificial intelligence, could complement and enhance the learning experience with 3D printing in the educational context.

These future directions can not only enrich our current understanding of 3D printing implementation in mathematics education but also provide new tools and strategies to improve the quality and equity of STEM learning overall.

CONCLUSIONS

The application of STEM education, with a focus on 3D printing in teaching Platonic solids, offers a promising path to enhance students' understanding and engagement in university mathematics education. The research and case studies presented support the effectiveness of this methodology, but continuous efforts are needed to address logistical challenges and ensure successful and equitable implementation.

The bold fusion of STEM education, mathematics, and art through the integration of 3D printing in teaching Platonic solids reveals an educational landscape full of possibilities and challenges. In this multidisciplinary journey, we have explored the transformation of abstract concepts into tangible entities, the rich intersection between science, mathematics, and art, as well as the technological and ethical challenges that this convergence implies.

The technological and pedagogical challenges inherent in 3D printing in STEM education cannot be underestimated. As Mishra and Koehler (2006) caution, "integrating technology into teaching is not simply adding electronic devices." It is crucial to address these challenges with adequate preparation, both in terms of technological competence and effective pedagogy. Investment in teacher training and technological infrastructure becomes imperative to overcome these barriers.

At the confluence of these disciplines and challenges, the integration of 3D printing in teaching Platonic solids emerges as invaluable preparation for an interconnected future. By equipping students with skills beyond pure mathematics, they are prepared to embrace the complexity of a world that demands interdisciplinary solutions.

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