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Impact of multiple representations-based instruction on basic six pupils' performance in solving problems on common fractions

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ABSTRACT

The study sought to investigate the impacts of multiple representations-based instructions (MRBI) on basic six pupils' performance in solving problems on common fractions in the Sagnarigu Municipality of Ghana. The study employed non-equivalent control group design in which 96 pupils in one primary school were purposively sampled and assigned to experimental group (n=46) and control group (n=50). Data were gathered using tests (pre- and post-test) and analyzed using descriptive and inferential statistics (independent samples t-test). Also, pupils' exposed to multiple representations-based instructions performed better in the post-test than in the conventional group. This suggests that MRBI positively impacted pupils' performance in solving problems on common fractions. We conclude that MRBI is an effective approach, which mathematics teachers need to incorporate in their teaching of fractions. Therefore, we recommend the MRBI approach for basic school mathematics teachers to enhance pupils' understanding of mathematics concepts, especially at mathematics education's foundation (basic level).

Keywords: multiple representations-based instruction, performance, common fractions, effects

INTRODUCTION

Finding ways to empower pupils to understand and do mathematics is at the heart of mathematics education reform (National Mathematics Advisory Panel, 2008). Learning or doing mathematics entails manipulating mathematical symbols and organising and interpreting mathematical relationships and conditions using specialized language, symbols, graphs, or other representations, resolving problems, deducing conclusions, and developing appropriate tools (National Research Council, 2006). An emerging theoretical view on mathematical learning is the use of multiple representations to establish links between graphical, tabular, symbolic, and verbal explanations of mathematical relationships and problem situations during learning and teaching to encourage and help pupils develop an understanding of mathematical relationships and concepts (Ainsworth, 2006; Uttal & O'Doherty, 2008).

When it comes to mathematics learning and teaching, the concept of multiple representations can take a variety of meanings. Flores et al. (2019) describe multiple representations as any mental state with a specified content, a mental replication of a prior mental state, an image, a symbol, or sign, or symbolic instrument that must be learned, or something that takes the place of something else. Lesh et al. (1987) acknowledge that representations are crucial for understanding mathematical concepts. Representation, according to Lesh et al. (1987), is an "external (and therefore observable) embodiments of pupils' internal conceptualizations" (p. 34). This means that for pupils to be able to understand a mathematical concept, they should have the ability of making translations within and between modes of representations.

Lesh et al. (1987) have come out with five modes of representations that occur in mathematics learning and problem solving. They are:

(i) "pictures or diagrams (static figural models),
(ii) written symbols,
(iii) spoken symbols,
(iv) manipulatives, and
(v) real-world situations where knowledge is organized around "real-world" events" (p. 38).

Multiple representations, according to Kaminski and Sloutsky (2012a), are external mathematical manifestations of ideas and concepts that communicate the same information in multiple forms. For example, "difference quotients, slopes of graphs in the coordinate plane, or formal algebraic derivatives" are all examples of the rate of change (Porzio, 1999, p. 1). In view of these definitions, multiple representations can be described as a means of displaying the same data in more than one form of external mathematical representation.

Multiple representations is one of the primary components in the mathematics curriculum that should be emphasized throughout the learning and teaching process (National Council of Teachers of Mathematics [NCTM], 2000). Pupils who can translate within and among numerous representations of the same problems or mathematical concepts stand the chance of having more flexible problem-solving tools and a greater appreciation for mathematics' consistency and beauty. Combining text-based (e.g., word problems) and symbolic (e.g., equations) representations with visual representations (e.g., graphs and other drawings) can help pupils learn more successfully (Rau, 2016). Classroom teachers usually use multiple visual representations to stress complementary conceptual components. This is because no single visual depiction can fully convey the uncertainty of mathematical concepts. Nevertheless, the relationships between visual representations and their referents are usually ambiguous, making learning difficult (diSessa, 2004; National Research Council, 2006).

The representation challenge is complicated by these issues (Rau, 2016). Visual representations must be used to teach pupils domain knowledge. Rau and Matthews (2017) opine that pupils could understand from the line graph that the distance a particle moves in a given period increases as time advances. The visual representations will be taught to the pupils and their relationship to the referent. When learning about the graph, they must comprehend what each axis represents and coordinate the two. When studying mathematics, the representation conundrum poses a significant educational challenge. Pupils are frequently forced to learn about unexpected subjects through unusual visual representations (Rau et al., 2015). Pupils will require representational competencies to overcome this problem: knowledge and skills to reason about and solve problems using visual representations (National Research Council, 2006).

Fraction is one concept that these pupils struggle with, negatively impacting their overall math ability. Fractions are essential component of the mathematics curricula from elementary school through to the high school. Canterbury (2007) asserts that, most pupils encounter problems in learning fraction concepts due traditional fraction instruction, and that to overcome these difficulties, instructional approaches that emphasize conceptual knowledge rather than procedural knowledge are required. Mathematics facilitators at almost all levels focus primarily on using the algebraic system of representations, ignoring other representations such as geometric or intuitive, for the simple reason that the algebraic system of representing fractions is formal while the others are not (Hitt, 2002). As a result, when teaching complex ideas like fractions, they are only related to limited representations. This is due to the fact that they believe the algebraic system of representations is formal while the others are not. Some pupils' troubles understanding mathematically complicated ideas, such as fractions, could be linked to limited representations in the classroom.

Mathematical concepts are built through conceptual knowledge, which is dependent on the usage of numerous types of representations, manipulations, and communication between these representations. The concept will be better, deeper, and last longer if more representations are employed (Pal, 2014). According to Ainsworth (2008), if all information about a complicated topic is included in a single representation, the representation is likely to distort it. Because one representation cannot adequately explain a mathematical concept as each representation has its own set of advantages; and therefore, the usage of multiple representations provides foundation for mathematical understanding. In fraction teaching, the utilization of a variety of non-traditional instructional methodologies and activities aids pupils in making efficient conceptual knowledge linkages (Canterbury, 2007).

Research has established that visual representations significantly impact how pupils' conceptualise fractions (Akkus & Cakiroglu, 2009; Charalambous & Pitta-Pantazi, 2007; Dlamini, 2017). Learners can benefit from varied representations once they have mastered numerous complex activities, according to research that has looked into the effectiveness of such environments. In related research, Morales et al. (2020) in their study looked at the impact of two teaching methods (multiple-representation instruction [MR] and conventional algorithmic instruction [TA]) on pupils' performance in fractions, decimals, and percent problems. The effects of the educational treatments and their sequencing on pupil performance were compared using a 2×2 crossover design. Pupils in the MR/TA treatment series received multi-representation instruction followed by traditional algorithmic teaching, while pupils in the TA/MR treatment sequence received treatments in the opposite order. A total of 89 seventh-graders enrolled in a pre-algebra class at an urban middle school in the Midwest took part in the study. Although the typical algorithmic treatment had a significant impact, the results showed that both instructional methods (MR and TA) improved performance. There was no significant difference in performance based on the order of teaching treatments.

The diverse representations of a complex idea, such as fractions, typically reveal distinct elements of the concept and make it easier to understand. UKEssays (2018) noted that traditional style of instruction (chalk and talk) generates fear, anxiety, unease, and insecurity in many pupils. This has precipitated a growing body of research in mathematics education to give importance to the role multiple representations play in mathematical learning and teaching (Rau & Matthews, 2017; Siemon et al., 2015). Rau and Matthews (2017) for example noted that visual representations employ by teachers helps promote mathematical reasoning among pupils. There is therefore the need for teachers to use variety of representations to help pupils understand difficult mathematics concepts (such as fractions) since no single representation can depict all aspects of a mathematical concept. Multiple representations can assist pupils in overcoming some of the obstacles they face when addressing problems by providing multiple concretizations of concepts.

On the other hand, opponents of multiple representations argue that using multiple representations in the classroom tends to hinder pupils comprehension and ability to solve problems rather than to help improve their conceptual understanding

(Kaminski & Sloutsky, 2013; Kaminski et al., 2013). They noted that if numerous representations are not employed correctly, they may fail to improve pupils' learning. Using many representations, for example, may confuse rather than help pupils learn unless they can:

(i) accurately understand each representation, and
(ii) connect various representations to the information they are trying to impart.

Teachers' pedagogical knowledge is important since teaching techniques affect pupils' grasp of concepts. Instead of employing the appropriate pedagogical content knowledge, many teachers teach from their prior knowledge (Valles, 2014).

Part-to-whole relationships are represented by fractions. Fractions are considered as one of the most challenging topics in basic school mathematics curriculum (Van Galen et al., 2008). Studies (e.g., Muzheve & Capraro, 2011; National Mathematics Advisory Panel, 2008; Siemon et al., 2015) indicate that many pupils have difficulties in learning the concept of fractions. For example, Muzheve and Capraro (2011) in their study among pupils in grade 6 to grade 8 found that pupils hold misconceptions with fractions. The chief examiners' report for basic education certificate examination (BECE) (Chief Examiner's Report, 2015, 2016, 2017) consistently indicated that pupils have difficulty solving fractions-related problems. The reports pointed out that pupils who answer fractions-related problems usually perform poorly because they lack the basic mathematical understanding that inhibits their problem-solving ability, affecting their general mathematics performance. The chief examiners recommend that teachers should employ multiple representations-based instructions to help pupils understand the underlying concepts of fractions (Chief Examiner's Report, 2017). A close looked at classwork and homework of pupils in Kamina Basic School confirmed pupils' difficulty in solving problems in fractions.

Among the reasons for pupils struggle in learning fractions could be ascribed to the pedagogical decisions teachers make towards the teaching of fractions. In other words, the kind and quality of mathematical representations employed by teachers to help pupils learn the concept of fractions. The teaching of complex subjects like fractions should:

(i) demonstrate conceptual understanding,
(ii) encompass a number of different representations of the concept, and
(iii) give pupils opportunities to solve problems in real-life and mathematical world contexts (NCTM, 2000).

Rau and Matthews (2017) assert that teachers generally introduce the topic of fractions by explaining it as a part-whole relationship, providing examples, and then following it up with a set of rules (algorithms) for solving problems related to fractions. Teachers who teach fractions using these strategies only succeed in helping pupils understand the rules or algorithms, but not the concepts and processes.

To help pupils overcome these difficulties in learning fractions, National Council for Curriculum and Assessment (NaCCA) (2019) recommends the use of multiple representations as instructional strategy/technique in the Ghanaian classroom. In spite of the recommendations for the use of multiple representations to improve learning and teaching of mathematics and fractions in particular, there appears to be no or little studies on the effect of multiple representations-based instruction in learning and teaching of mathematics especially fractions. There is therefore the need to create awareness among mathematics teachers through research on the impact of multiple representations-based instruction on the pupils' performance in fractions and mathematics at large. The findings from this study would add to the growing body of research demonstrating the effectiveness of multiple representations-based instruction practices as it could offer significant scientific acquisition in understanding related fractions and its related concepts. In view of this, we sought to investigate the effects of a treatment based on multiple representations on basic six pupils' performance in solving problems on common fractions in the Sagnarigu Municipality of Ghana? Specifically, the study attempts to answer the research question: Does multiple representations-based instruction as a treatment affect basic six pupils' performance in solving problems on common fractions in the Sagnarigu Municipality? In view of this, the study also sought to test the null hypothesis "There is no statistically significant difference in the mean performance scores between the experimental and the control groups." And the alternative hypothesis "There is a statistically significant difference in the mean performance scores between the experimental and the control groups."

METHOD

Research Design

We employed non-equivalent control group design to investigate whether multiple representations affect basic six pupils' performance in solving problems on common fractions. The amount to which researchers effectively establish equivalence between the experimental and control groups indicates the validity of the design (Campbell & Stanley, 1963; Toulany et al., 2013). According to quasi-experimental researchers, randomisation may fail to establish precisely similar groups regarding crucial features to the study's outcome. In such cases, the researcher employs a non-equivalent control group design. To this effect, we picked two comparable classrooms of the same school in this study.

Sample

The study was conducted in Kamina Primary School in Sagnarigu Municipality in the Northern Region of Ghana. In this study, one school (Kamina Primary School) was selected as the sample, with class six 'A' as the experimental group and class six 'B' as the control group. The classes are both intact groups of Kamina Primary School. Class six 'A' had 46 pupils, and class six 'B' had 50 pupils. In all, 96 of the population was used. The respondents of this study ranged in age from 11 to 13. The details are summarized in **Table 1**.

Table 1. Demographic characteristics of pupils in the experimental and control groups

Groups	Male	Female	Total
Experimental–class “A”	19	27	46
Control–class “B”	27	23	50
Total	46	50	96

Table 2. Scoring rubrics

S/N	Comments	Marks
1	Correct mathematical representation	B1
2	Right method	M1
3	Correct answer	A1

Validity and Reliability of Instruments

The pre- and the post-test consisted of 10 problems each on problem-solving on common fractions. The questions were structured in accordance with the primary six mathematics curriculum (NaCCA, 2019). In our quest to determine the face and content validity of the test instruments, we presented the instruments to one professor of mathematics education and two experienced basic school mathematics teachers to seek for their expert judgement.

A pilot study of the instructional design and the test instrument was conducted among 30 pupils from a selected school in the same community in which the actual study took place. The selected schools for the pilot test had similar features as that of the one sampled for the study. This was done to collect information which would comment or relate to the instruction and the procedure, as well as suggesting criticisms and possible improvements. There were 15 participants in each of the groups: experimental and control groups. The pilot study was done for a week, and both groups were given a pre- and a post-test. A Cronbach alpha statistic was used to test the reliability of the fraction test items for both the pre-test and post-test; and these yielded alpha value of 0.72 and 0.76, respectively. These are considered as acceptable reliability for the scales (Taber, 2018).

Issues of Internal and External Validity

All the pupils in the experimental and control groups received a pre-test before the treatment, and as a result may pose a threat to internal validity. It is believed that pupils who receive pre-test in a study may react somehow strongly to the treatment than they would have done if they fail to take the pretest (Cikla, 2004). In view of this, both groups were pretested beforehand so as to minimize the threat. Also, since the study was conducted in only one school of two streams, there was a threat posed by location which needed to be acknowledged. However, no outside events affecting participants’ responses were notified during the administration of the instruments. In both groups, almost all of the pupils participated fully from the pre-test administration through to the post-test. Hence, there were no record of missing data in the pretests and posttests. In one of the instructional sessions, three of the pupils did not join the class due to reason best known to them. The study was conducted in the same school in different classrooms under similar conditions. The conditions in the two classrooms were more or less the same, the size of the classes were almost the same as well as the sitting arrangements and the lighting. In view of this, we can confidently say that the threats to the ecological validity were also contained.

Procedure

Each question carried three marks. A total of 30 marks were allocated to both the pre-test and the post-test. In order to facilitate the scoring of the pupils’ responses to the tests, marking rubrics were developed to score each question a maximum of three marks as depicted in **Table 2**.

Prior to the commencement of the instrument administration, we visited the selected school to seek permission to carry out the study in the school from the Headteacher. After permission was granted, we met the pupils in the selected classes in their respective classrooms to acquaint them with the study and address any concerns which they could have concerned the study. Afterwards, we sought their consent, and fix the date for the administration of the instrument and the instruction.

Two types of lesson plans on the addition, subtraction and multiplication of common fractions were designed for the control and experimental groups by both authors. One of the authors taught the control group using the conventional approach. The lesson was intentionally developed to reflect this conventional method. The other author also taught the experimental group to minimise the issue of bias using multiple representations-based instructions. The administration of the instructions and tests in both experimental and control groups were done concurrently.

On the first day of the first week of implementation, a pre-test was conducted for both experimental and control groups (see **Appendix A**). Instructions and testing were conducted in regular classroom settings during the regular lesson periods. The instructional processes took six sessions with three in every week. Each of the sessions lasted nearly 35 minutes. During the instructional sessions, the experimental group underwent an intervention to learn the addition of common fractions through multiple representations. We employed external representations such as paper folding/shading, number line model, symbols, and verbal representations (Ball, 1993), to affect the internal representations of the fraction concept. On the other hand, the control group learned the addition of common fractions through conventional approach where lecture and discussions were employed. To learn how to solve problems involving common fractions as spelled out in the basic six curriculum (NaCCA, 2019), we reviewed the groups’ concept of common fractions from constructing to comparing fractions before operations on addition, subtraction and multiplication involving common fractions.

Table 3. Group statistics of pupils' scores in the pre-test

	Group of pupils	n	Mean	Standard deviation	Standard error mean
Scores	Experimental	46	6.37	2.42	0.36
	Control	50	6.26	2.66	0.38

Table 4. Group statistics of pupils' scores in the pre-test

	LT		t-test for equality of means						
	F	Sig.	T	df	Sig. (2-tailed)	MD	SED	95% CI of difference	
								Lower	Upper
Scores Equal variances assumed	0.00	0.98	0.21	94	0.83	0.11	0.52	-0.92	1.14

Note. LT: Levene's test for equality of variances; MD: mean difference; SED: Standard error difference; & CI: Confidence interval

Table 5. Group statistics of pupils' scores in the post-test

	Group of pupils	n	Mean	Standard deviation	Standard error mean
Scores	Experimental	46	20.13	6.38	0.94
	Control	50	11.74	5.63	0.80

In the first instructional session, pupils from the experimental group learned how to construct common fractions using multiple representations whereas, pupils from the control group learned how to construct common fractions using conventional method of teaching. Following that, pupils participated in instructional activities involving the construction and comparison of common fractions. In the second instructional session, pupils were taught how to solve addition of common fraction tasks and related word-problems through multiple representations-based instruction (using paper folding/ shading, number line model, symbols, and verbal representations). The instruction on addition of common fraction continued the third day.

In the fourth session, pupils were given instruction on how to solve problems involving subtraction of common fraction tasks. The instruction on subtraction of common fraction continued the fifth day. In the six session, pupils were now taught how to solve problems involving multiplication of common fraction. On the following day, pupils from the control and experimental groups sat for the post-test (see **Appendix B**).

RESULTS AND DISCUSSION

Results

Does multiple representations-based instruction as a treatment affect basic six pupils' performance in solving problems on common fractions in the Sagnarigu Municipality?

Research question 1 sought to determine primary school pupils' performance in solving problems on common fractions. Pupils were first given a set of questions to solve as a pre-test. The pre-test comprises 10 questions marked out of 30. This test was administered to 96 pupils.

A pre-test was conducted to ascertain the performance of the experimental and control groups in solving problems on common fractions prior to the treatment. In view of this, an independent t-test was employed to compare the performances of the experimental and control groups to establish their equivalence in terms of their performance in solving problems on common fractions. **Table 3** provides useful descriptive statistics of the mean scores for the two groups (experimental and control groups).

The examination of the group means in the pre-test from **Table 3** indicates that, the experimental group obtained a mean of 6.37 and standard deviation of 2.42 whereas, the control group obtained mean of 6.26 and standard deviation of 2.66.

The independent samples t-test results in **Table 4** helps to investigate whether the observed difference in mean scores obtained by the control and experimental groups are significant or not after the treatment using an alpha level of 0.05. **Table 4** shows the variability of means between the two groups.

In the pre-test, the t-test for independent samples findings in **Table 4** shows no significant difference in the scores obtained by the experimental group ($M=6.37$, $SD=2.42$, $n=46$) compared to the control group ($M=6.26$, $SD=2.66$, $n=50$), with $t(94)=0.21$, $p<0.83$. As a result, we conclude that the two groups (experimental and control) are similar in terms of their performance in solving problems on common fractions.

Null hypothesis: There is no statistically significant difference in the mean performance scores between the experimental and the control groups

Alternative hypothesis: There is no statistically significant difference in the mean performance scores between the experimental and the control groups

The post-treatment test was conducted to ascertain the performance of the experimental and control groups in solving problems on common fractions after the treatment. In view of this, an independent t-test was employed to compare the performances of the experimental group and the control group to establish the effect of multiple representations-based instructions on pupils' performance in solving problems on fractions. **Table 5** provides useful descriptive statistics of the mean scores for the two groups (experimental and control groups).

Table 6. Group statistics of pupils' scores in the post-test

	LT		t-test for equality of means						
	F	Sig.	T	df	Sig. (2-tailed)	MD	SED	95% CI of difference	
								Lower	Upper
Scores Equal variances assumed	0.61	0.44	6.85	94	0.00	8.39	1.23	5.96	10.82

Note. LT: Levene's test for equality of variances; MD: mean difference; SED: Standard error difference; & CI: Confidence interval

From **Table 5**, the examination of the group means indicates that experimental group (M=20.13, SD=6.38) performed better in the post-treatment test than did control group (M=11.74, SD=5.63).

The independent samples t-test results in **Table 6** helps to investigate whether the observed difference in mean scores obtained by the control and experimental groups are significant or not after the treatment using an alpha level of 0.05. **Table 6** shows the variability of means between the two groups.

In the post-treatment test, the t-test for independent samples findings in **Table 6** demonstrated a significant difference in the scores achieved by the experimental group (M=20.13, SD=6.38, n=46) compared to the control group (M=11.74, SD=5.63, n=50), with t(94)=6.85, p=0.00. As a result, we reject the null hypothesis and conclude that the mean performance scores of the experimental and control groups differ significantly. The performance scores of the control and experimental groups had a large effect size (d=1.40). This indicates that multiple representations-based instructions greatly impact children's performance in solving problems on common fraction. In light of this findings, we argue that since the experimental and control groups had similar scores in the pre-test and different scores in the post-test, one can be more confident that changes in the post-test scores was due to MRBI and not to othere related factors.

Discussions

In our contemporary world today, the use of diverse external representations in learning and teaching of mathematics appears to have recently received general acceptance (NCTM, 2000, 2006). Several studies (Ainsworth, 2008; Akkus & Cakiroglu, 2009; Kaminski & Sloutsky, 2012b) have emphasised the need to employ various representations or models to support and assess pupils' fraction constructs. The representational systems are central for conceptual learning and dictate what is learned to a considerable extent (Kaminski & Sloutsky, 2012a). Learning takes the form of diagrams, practical demonstrations, abstract mathematical models, simulations, and other methods to represent information (Schuyter & Dekeyser, 2007). For pupils in basic school, fractions are one of the most difficult mathematical concepts to understand (Mendiburo & Hasselbring, 2011). This is confirmed by the study's findings, which revealed that pupils exhibited poor performance in solving problems on common fractions.

The ability to identify the same idea in numerous representation systems, alter the concept within these representations, and transfer the concept flexibly from one representation system to another is required for concept acquisition and enables pupils to identify rich connections (Cikla, 2004). According to Canterbury (2007), pupils will develop conceptual comprehension of fractions by using a variety of non-traditional fraction instructional approaches and activities in fraction lessons. For example, each fraction notion can be represented to pupils in many ways, including written symbols, written language concrete models, and diagrams, which is what many representations are all about. Indeed, numerous theoretical and empirical works (e.g., Ainsworth, 2006; National Research Council, 2006; Rau, 2016; Uttal & O'Doherty, 2008) encourage the use of multiple representations, meaning that combining visual (e.g., graphs and other illustrations) with text-based (e.g., word problems) and symbolic (e.g., equations) representations can help pupils learn more effectively. They claimed that no one visual representation accurately portrays the complexity of mathematical concepts; instead, educators frequently employ many visual representations, each of which emphasizes various conceptual features to promote understanding.

The use of numerous representations in mathematics has been investigated in many scientific papers and the literature. The research found that teaching mathematical ideas through numerous representations helps pupils better understand the relationships between concepts and achieve a deeper knowledge. This supports Kaminski and Sloutsky (2012b) and Mohammed (2009) claim that teachers' ability to recognize and represent the same thing in different ways, as well as flexibility in moving from one representation to another, help the learners to see rich connections, improve conceptual understanding, broaden, and deepen one's understanding, and intensify one's ability to solve problems. Koedinger and Terao (2004) argue that, while pupils did well on the questions, "picture algebra" was not a cure for developing algebraic thinking in pupils because, after representing the algebraic context with diagrams, they still had difficulties translating this representational mode to the algebraic one to solve the equation. The researchers also stated that while employing visuals may help pupils learn mathematics, it is not a way that reaches every learner to assist them in understanding algebraic sense.

We conclude that by using an instructional strategy (such as MRBI) that provides pupils with access to and learning different representations of relations and concepts, pupils' difficulties in solving problems on common fractions can be reduced to some extent.

Limitations of the Study

Among the limitations of this study is the role of authors as classroom teachers. Participants in both experimental and control groups were instructed by the two authors with each one assigned to a group. Although, we tried to remain unbiased as much as possible but there may still be some little personal biases and enthusiasm that may influence the study results.

Another limitation of the study is the failure to randomly assigned participants to groups. Since participants were not randomly assigned to groups, there is likelihood that pre-test results may though show that the groups are similar with respect to the dependent variable but not that they are similar with respect to other variables such as experiences, age variability, etc.

Moreover, despite the fact that instructors in both groups have some commonalities in terms of their background, the way in which each author performed their instruction could be important factors to consider.

CONCLUSION

The study revealed that pupils exposed to the multiple representations-based instructions performed better in the post-test as against pupils of the conventional group since MRBI as a teaching strategy was more effective in solving problems in common fractions than the conventional approach. We argue that although there was absence of random assignment, for the fact that the groups were measured before and after treatment, there can be strong confidence in the similarity of the groups. And that one can conclude that the instructions was to a larger extent responsible for the differences observed in post-test.

The superiority of the MRBI strategy over the conventional approach could be attributed to diagrams, graphs, symblos and other visual representations in the teaching and learning of fractions. Pupils who were exposed to this type of strategy are more likely to build their internal representation of the concept, which will help improve their external representations, increasing their content knowledge and a general improvement in their mathematics performance. In view of this, we recommend the MRBI approach for basic school mathematics teachers to enhance pupils' understanding of mathematics concepts, especially at mathematics education's foundation (basic level). We acknowledge the fact that this study was delimited to common fractions. Hence, future research can be conducted beyond this scope to include other algebraic contents.

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Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Pre-Test

(1) Arrange these fractions in ascending order: $\frac{3}{4}, \frac{2}{3}, and\ \frac{5}{6}$.

(2) Add $1\frac{1}{4} + \frac{3}{4}$.

(3) Subtract $1\frac{2}{5} - \frac{4}{5}$.

(4) Multiply $\frac{5}{8} \times 3\frac{1}{5}$.

(5) Tiyumba ate $\frac{4}{7}$ of a cake. Find the fraction of the Cake left.

(6) Ama used $\frac{2}{3}$ of flour to bake bread and $\frac{1}{6}$ pounds to bake cake. What fraction of flour did she use for baking?

(7) $\frac{5}{8}$ of the teaching staff in a school are male. What fraction of the teaching staff is female?

(8) There was $\frac{7}{9}$ of a pie left in the fridge. Danso ate $\frac{1}{4}$ of the leftover pie. How much of a pie did he have?

(9) Mr. Jebuni spent 1/5 of his salary on utilities and the rest on food. How much did he spend on food?

(10) Grace gave 2/5 of flour to Hagar and 1/3 to Titus. What quantity of flour did she give out?

APPENDIX B

Post-Test

(1) Arrange these fractions in ascending order: $\frac{7}{12}, \frac{5}{6}, and\ \frac{3}{4}$.

(2) Add $1\frac{2}{5} + \frac{4}{5}$.

(3) Subtract $2\frac{1}{6} - \frac{5}{6}$.

(4) Multiply $\frac{3}{5} \times 1\frac{4}{7}$.

(5) In a class, $\frac{1}{5}$ of the students speaks English and $\frac{1}{3}$ of them speaks Dagaare. What fractions of the class do not speak either English or French?

(6) There are $\frac{7}{9}$ kilograms of salt in the kitchen. Mr. Akurugu used $\frac{3}{11}$ of the salt when he was preparing dinner. How much salt did he use?

(7) A school caterer had $\frac{1}{2}$ gallon of tomatoe sauce she put $\frac{2}{6}$ gallon of sauce in her beef stew. How many gallons of tomato sauce is she left with?

(8) John's and Tina's parents both drove from Savelugu to Kamina. John's father drove $\frac{1}{5}$ of their way to school whilst Tina's mother drove $\frac{2}{5}$ of the way to school. How much of the way did they drive altogether?

(9) A bread seller used 2/4 pounds of flour to bake bread and ½ pounds of flour to bake pizza. How many pounds of flour did she use altogether?

(10) Ayele drank 1/5 of a bottle of apple juice. How much of the carton of apple juice is she left with?