**Research Article** 

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## Teaching approaches of the functions in seventh grade, the Kosovo case

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ARTICLE INFO	ABSTRACT
Received: 13 Nov 2024	In everyday life and mathematics, we often deal with connections and relationships between objects of the same
Accepted: 09 Jan 2025	type or different types. Thus, we associate the elements of a community or a set in some way with the elements of the same community (set) or of another community (set). For example, we assign a grade to each student, a price to each commodity, a corresponding number to all pages of a book, and so on. Students learn the functions almost every school year, starting with the seventh grade and so on. There are different ways of explaining functions so that students can more easily understand their meaning in achieving results. In the time we are living thanks to the advancement of technology, we are having the opportunity to search for different ideas on the Internet for the explanation and development of various activities related to daily life. Illustrations greatly influence students to have as much concentration as possible during the lesson and in terms of the implementation of functions In this paper, we have tried to clearly explain the meaning and application of linear functions as well as their graphical presentation. Also, in this paper we have tried to illustrate with some examples the graphica representation of linear functions, giving different practical techniques through different figures and visualizations. A questionnaire in this paper included 88 seventh-grade students at the school "Elena Gjika" city Prishtina, Kosovo. With the help of the statistical test, we have concluded that practical techniques such as illustrations, various visualizations, and practical work are important in learning the function and its graphica presentation.
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## INTRODUCTION

Functions constitute an important part of mathematical literacy. For more than twenty years, the concept of function has been internationally considered a unifying theme in mathematics curricula (Steele et al., 2013). Hence, "change and relationships" is one of the four content subscales in mathematics as defined by the Programme for International Student Assessment (PISA) of the Organization for Economic Co-operation and Development (OECD, 2017).

While working with students as teachers over the years we have encountered different practices for explaining functions. Kieran (1992) questions whether students' inability to conceptually understand functions is related to their teaching or is due to students' inappropriate way of approaching function tasks. There are many methods to explain and illustrate by concrete techniques using different figures and visualization methods so that students can more easily understand, making the lesson and explanation more attractive. Through visualization, any organization can be synoptically grasped as a configuration. In this way, we have as many kinds of visualization as kinds of units: geometrical configurations where units are ID or 2D shapes or Gestalts, Cartesian graphs where units are couples {point, coordinates}, and propositional graphs where units are statements. For the visualization of each register of visualization, there are some rules or intrinsic constraints to produce units and form their relations. Thus, geometrical configurations can be constructed with tools and according to the mathematical properties of the represented objects (Duval, 1999). Janvier (1987) states that the use of representations in mathematical thinking is fundamental and most of the textbooks today make use of a wide variety of diagrams and pictures to promote mathematics understanding. The technology and the time we are living make this easier because we can also use the resources from the internet for additional lessons and see various illustrations and videos that explain the meaning of fractions, and operations with them more understandably and attractive way for the students. It helps them a lot to obey the rules and to avoid mistakes (Kamberi et al., 2022). Vinner and Dreyfus (1989) focus on the influence of concept images over concept definitions. Except for illustrations, an important role in understanding play tasks, classroom commitment, extra hours, and extra exercises (Aliu et al., 2021). Sajka (2003) indicates that students' abilities in solving tasks involving functions are influenced by the typical nature of school tasks, leading to the use of standard procedures. According to a standard didactic sequence, students are asked to infer the properties of a function using the

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given graph, by following a specific procedure (Panaoura et al., 2015). The illustrations have a greater impact compared with using the program GeoGebra and Mathematica for solving exercises by students than solving exercises on the whiteboard, since students of lower classes hardly find themselves working with GeoGebra, compared to higher classes (Mollakuqe et al., 2021). Specific attention was given to literature discussing the use of GeoGebra in the context of functions. The survey responses underscored that GeoGebra not only made complex mathematical concepts more accessible but also fostered a sense of engagement and curiosity among students. The hands-on experience with GeoGebra was credited with significantly enhancing students' understanding exponential growth compared to conventional classroom (Tuda & Rexhepi, 2024). approaches.

Elia et al. (2007) examined secondary pupils' conceptions of function based on three indicators:

- (1) Pupils' ideas of what function is,
- (2) Their ability to recognize functions in different forms of representations, and
- (3) Problem-solving involved the conversion of a function from one representation to another.

Findings revealed pupils' difficulties in giving a proper definition for the concept of function. Even those pupils who could give a correct definition of function were not necessarily able to successfully solve function problems.

Collaborative mentoring has helped us even more in our planning regarding our teaching. Seeing different examples of the lesson, we as teachers have made the difference between the classes where there were illustrations and those that did not exist at all. In the classes where various illustrations and demonstrations are practiced, students always have more concentration during the development of the unit and where there is interest there will be higher results.

## **METHODOLOGY**

This study was aimed to determine if practical techniques such as illustrations, various visualizations, illustrations, and practical work are easier to learn and understand the function and graphic presentation. For this purpose, the case study model, which is a qualitative research design, was used in the research. The case study is defined as the in-depth exploration of a specific system (e.g., an activity, event, process, or individuals) with the help of different data collection tools. Thus, to be able to deeply examine the understanding of primary school students of the concept of function, the case study model was chosen for this research. For this purpose, the first step is to search for relevant literature in the academic database and then filter this literature giving priority to peer-reviewed articles, books, and conference proceedings that provide in-depth insights into the theoretical foundation, graphical representation, practical applications, visualization tools associated with understanding of functions was included. The second step presents a comparison of the classes that used the modern techniques mentioned in the paper and the classes that used the classical standard methods. This was done through tests, interviews, and questionnaires, which we will elaborate on for participants and data collection below.

#### Participant and data collection and data analysis

In the paper, we use the table of specifics, where the main purpose was to discover the problems and difficulties that students have in learning and understanding the concept of the function. For this reason, this study was conducted at a state level with some 7<sup>th</sup>-grade students. To conclude modern methods (with special emphasis on illustrations, and visualizations) affect the understanding of the concept of function in the 7<sup>th</sup> grade, we have conducted tests in two different classes, in one where these methods are used and in the other where they were not used. For comparisons, we also conducted interviews with students of different levels of success.

We have also created a questionnaire to see how much students understand the functions and how important it is to use concrete modern tools to understand them.

In the analysis of the research data, a contingency table was used. We answered the answers to the questionnaire questions as variables X and Y, where the first variable has to do with whether the student followed the practical illustrations regarding the meaning of the function concept presented by the teacher and the second variable has to do with that if the students have understood the functions and operations with them. According to the answers made by the students, we conclude that these variables are dependent on each other.

#### **Understanding the Function**

Considering the curriculum and the age of our students, in this unit, we give some of our practices in the classroom in such a way that the concept of function is easier to understand, and the lesson becomes more attractive and catchier.

There are relationships between elements of different communities or relationships between elements of the same community. For example, in **Table 1** (Zejnullahu et al., 2004).

Table 1. Corresponding height and age of a child

		-					
Years of life	0	1	2	3	4	5	6
High (cm)	50	70	80	90	95	100	103

It is shown how (approximately) the height of the child varies depending on his years, from the day of birth (0 years) until the end of the sixth year. i.e. **Table 1** shows how the height of the child is a function of the years of life. This is just a simple example showing how one size is a function of another size.

There are many important examples of this type:

The surface area of a square is a function of the length of its side

The perimeter of a circle is a function of its radius

The surface area of a rectangle is a function of its dimensions

The path traveled (in cm) of the body that falls freely is a function of the time (in seconds) elapsed from the beginning of the fall.

In the following, we are giving a concrete example for the clearest possible understanding of the function.

**Example 1:** Jeton's salary as a dance instructor depends on the number of hours he works. If for one hour Jeton is paid 7 euros, what will be his salary for 8 hours, 15 hours, and 25 hours?

Table 2 shows the relationship between the number of hours Jetoni worked and his salary.

Table 2. Corresponding work hours and salary of Jetoni

Number of hours (argument, input values) <i>x</i>	Rule (function) f Multiply by 7	Salary earned (Output values y)
8	8 · 7	56
15	15 · 7	105
25	25 · 7	175

If we denote by x the input values, and by y, the output values, we can say that: for each value of x, by the rule, we associate one and only one corresponding value y. In this case, we say that y is a function of x since its value depends on the change in the value of x. We note:  $\gamma = f(x)$  (reads: "y is equal to f of x").

The input value *x* is called the independent variable or argument, while the output value *y* is called the dependent variable or function value.

The function *f* can also be represented by the set of ordered pairs (x, f(x)) where the first component is the input value or the independent variable, while the second component  $\gamma = f(x)$  is the output value, or the value of the function at the point.

Thus, the function in **Example 1** can be denoted:

$$f = \{(x, y): y = 7 \cdot x\} = \{(8,56), (15,105), (25,175)\}$$
(1)

Example 2: Is price a function of the number of items? Explain.

Four items can cost  $\in$  2 or  $\in$  3 (input value 4, corresponds to more than one output value). Thus, price is not a function of the number of items (**Table 3**).

Table 3. Corresponding number of items and their cost

Number of items (input value)	4	9	1	8	4
Price (output value)	2€	4€	0.50€	4€	3€

Despite the significant impact of tablets, GeoGebra is a valuable pedagogical tool for visualizing a wide variety of mathematical formulae in both algebraic and geometric representations (Mollakuqe et al., 2021). Through GeoGebra, we can visualize, analyze, and understand function representation like never before. This software not only allows us to observe the behavior of exponential functions but also experiment with various parameters, transforming, and translating these functions to gain a more profound insight into their properties (Tuda & Rexhepi, 2023).

**Example 3:** Determine the position of points *A* (5,4) and *B* (-2, -3) (Figure 1).

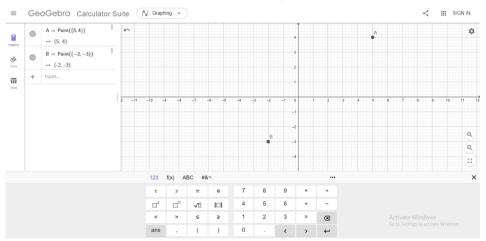


Figure 1. Using the Geogebra program to represent points in the coordinate system (Source: www.geogebra.org)

The graph of the numerical function is the set of all points of the plane Oxy that have as abscissa the x values of the function definition community, while ordinates have the corresponding values of the function f (x) (Shehu et al., 2019).

**Example 4:** The noise detector is an apparatus used in oceanography to determine the location of underwater objects. The operation of this apparatus is based on the echo of sound waves.

The formula  $\gamma = \frac{x}{2}$ , where x takes time for the echo to return to the ship, represents the distance from the surface where the object is located.

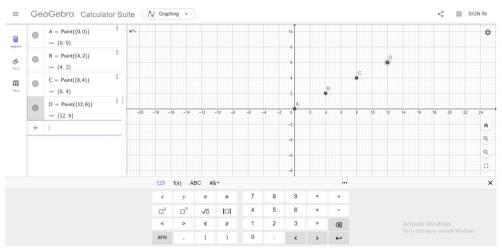
In the formula  $\gamma = \frac{x}{2}$ , x is the input value and y are the output value.

The rule (function) is  $\frac{x}{2}$ . We will now use the coordinate system to represent the graph of this function.

**Step 1**: If you choose input values 0, 4, 8 and 12, find the output values, then mark them as ordered pairs (**Table 4**). **Table 4**. Input and output values of a function

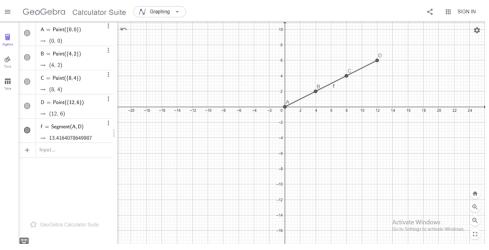
Input values	The rule (The function)	Output values	Pairs lined up
ĸ	$\frac{x}{2}$	Y	(x, y)
)	0	0	(0,0)
4	$\frac{4}{2}$	2	(4,2)
3	$\frac{8}{2}$	4	(8,4)
2	$\frac{12}{2}$	6	(12,6)

Step 2: Display the pairs listed by the table in the coordinate system Oxy (Figure 2)



**Figure 2.** Using the GeoGebra program to represent points in the coordinate system (x-coordinates represent the time it takes the echo to return to the ship; y -coordinates represent the distance from the surface where the object is located) (Source: Authors' own elaboration)

Step 3: Draw the line joining the points shown in the coordinate system (Figure 3).



**Figure 3.** Using GeoGebra program for graphical presentation of the function (Source: Authors' own elaboration, using www.geogebra.com)

The line represents the graph of the function  $\gamma = \frac{x}{2}$ .

#### Some Figures of Students during the Graphic Presentation Activities of the Functions

In the following, we will present some of the students' photographs with their works during the learning of the functions and their graphic presentation. These working methods make it much easier for students to learn the functions.

In **Figure 4** in honor of the holiday of November 28, the day of the flag, the students have first found the coordinates of the points for the words "*CONGRATULATIONS NOVEMBER 28*" and for the eagle. Then they presented the found points in the coordinate system and finally colored them. It was a job with great concentration, but the completion was very nice.



**Figure 4.** "Congratulations November 28" and national flag displayed in coordinate system (Source: Evidence of student work on the graph of the function)

In **Figure 5**, **Figure 6**, and **Figure 7** we will present some figures where in this case the students were given the ready coordinates and by presenting them in the coordinate system at the end, they saw the figures that only everyone knows.

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Figure 5. Mickey Mouse displayed in coordinate system (Source: Evidence of student work on the graph of the function)

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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	(13, -12)		(-13, 6)	(11, -5)	(0, -7)	(1,8)	(-13, -12)	No. of Concession, Name			
(15, 13) $(-11, 3)$ $(-14, 3)$ $(10, 4)$ $(2, 4)$ $(1, 7)(16, -14)$ $(9, 2)$ $(-14, 2)$ $(1, 7)$	(14, -13)	Shape 6	(-13, 3)	(10, -5)	(-1, -7)	(4,7)		1 North House			
(16, 14) $(.9, -2)$ $(7, -3)$ $(.2, -5)(7, -14)$ Share 17 $(10, 0)$ $(.1, -4)$	(15, -13)	(-11, -3)	(-14, 5)	(10, -6)	(-2, -6)	(1,7)					
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1.1.1.1 1.1.1.1 1.1.1.1 1.1.1.1	(7, -14)		Shape 17	(10, 0)	(-1, -4)			X			
(4,-15) (-13,2) (0,-4)					(0, -4)						
(1,-13) (-15, 4)	(1, -13)		(-15, 4)								

Figure 6. Snowman displayed in a coordinate system (Source: Evidence of student work on the graph of the function)



**Figure 7.** Graph of the function in the coordinate system, class VII1, VII2, teacher Saranda Kamberi (Source: Evidence of student work on the graph of the function)

# The Influence of Practical Illustrations, Visualizations, and Instructional Videos on Understanding the Function and Its Graphic Presentation

We tried to make a short questionnaire (**Figure 8**) with the 7<sup>th</sup>-grade students to see how well they understood the functions and whether they found graphic presentation easy. A total of 84 students participated in the questionnaire, where in the answer to the first question in the YES option a total of 56 students answered while with NO 28 students, while in the answer to the second question 42 students answered YES and 42 students answered NO (**Table 5**). In this questionnaire, the first question with YES is answered in total. In the following, we will elaborate on their answers.

#### Questionnaires

- Have you followed the practical illustrations regarding the meaning of the function concept presented by the teacher?
   A. Yes
  - B. No

## 2. Do you understand the functions and operations of them?

- A. Yes
- B. No

Figure 8. Questionnaire (Source: Authors' own elaboration)

Table 5. The results of the answers (Rexhepi et al., 2021)

	The answer to the first question									
The ensure to	X/Y	Yes	No	The total						
The answer to the second	Yes	14	28	42						
question	No	42	0	42						
	The total	56	28	84						

With the level of importance,  $\alpha = 0.01$  we will prove that the answers to the second question by the students do not depend on their opinion about the graphical representation of the function.

We first calculate the absolute expected frequencies for each of the quadrants in **Table 6** (Rexhepi et al., 2021). Based on the preliminary table, the expected values are in **Table 6**.

Table 6. The results of the answers (Rexhepi et al., 2021)

		The answer to the	ne first question	
The ensure to	X/Y	Yes	No	The total
The answer to the second	Yes	28	14	42
	No	28	14	42
question	The total	56	28	84

Next, we define the hi-square:

$$x^{2} = \frac{(14-28)^{2}}{28} + \frac{(28-14)^{2}}{14} + \frac{(42-28)^{2}}{28} + \frac{(0-14)^{2}}{14} = 42$$
(2)

With the level of importance  $\alpha = 0.01$ , we assign  $x_{1.001}^2 = 6.6349_{0.01}$ , the critical domain is  $C = (6.6349, \infty)$  and since 42 belongs to the interval  $C = (6.6349, \infty)$  we conclude that *x* and *y* are dependent.

To see how modern methods and illustrations affect learning, we have conducted a test in two classes, one where these methods and forms of work are used, as well as the class where they are not used. The test contains a total of 5 questions:

- 1. Definition of the function.
- 2. Find and mark the coordinates of the following figure.
- 3. Find the coordinates of the following figures.
- 4. By moving the points on the right side of the coordinate system, create the symmetry of the given figure.
- 5. Present the coordinates of the points in the coordinate system and join those points in order.

The students have the space to answer and practice the lessons about the functions where we have tried to always connect the lessons with real life during the lesson. **Figure 9** shows the model of the test conducted with the students.

Meanwhile, we are giving the test made by one of the students of the class where the illustrations and different work methods were practiced (**Figure 10** and **Figure 11**).

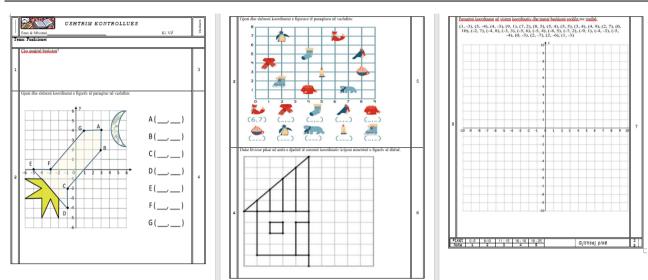
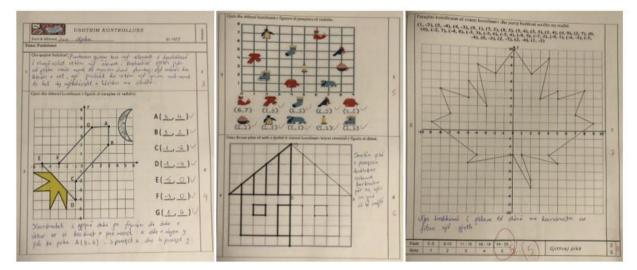


Figure 9. Test sample (Source: Evidence of student work on the graph of the function)



**Figure 10.** The test worked by the student who practiced the methods (Source: Evidence of student work on the graph of the function)

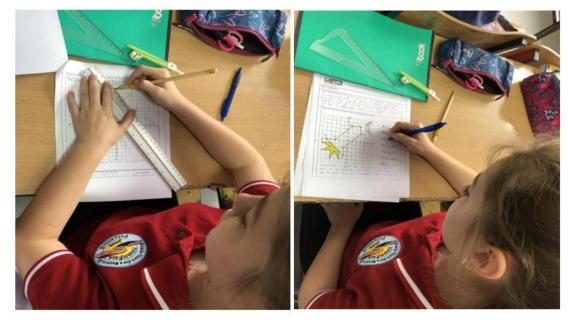
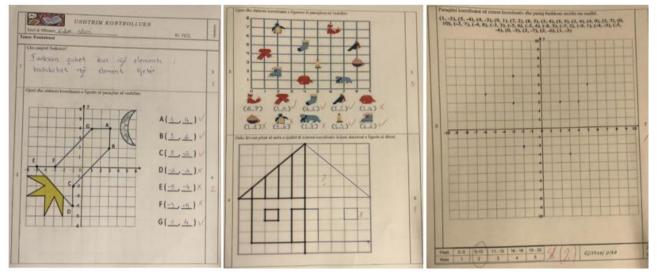


Figure 11. The student who practiced the methods (Source: Evidence of student work on the graph of the function)

## Answers:

- 1. A function is called when an element of a set is accompanied by one and only one element of another set, e.g. A student has her class, a product has a price it cannot have two at the same time, etc.
- 2. We find the coordinates by looking at the picture and knowing how the first coordinate is taken *x* and the second is taken *y* for example at point A (3,4), 3 represents *x*, and 4 represents *y*.
- 3. See Figure 10.
- 4. We present each point on the right in the coordinate system for as many units as it is on the left.
- 5. By joining the points in the coordinates, a leaf was obtained.

Figure 12 is of students from another class who did not practice the methods and illustrations for the functions.



**Figure 12.** The test worked by the student who did not practice the methods (Source: Evidence of student work on the graph of the function)

### Questions:

- 1. What is the definition of function?
- 2. Find and mark the coordinates of the figures presented below.
- 3. Find and mark the coordinates of the figures presented below.
- 4. By moving the points to the right side of the coordinate system create the symmetry of the given figure.
- 5. Present the coordinates in the coordinate system and then join each one in turn.

#### Answers:

1. A function is called when one element joins another element

It is clear from the students' work that the work we do is very important and the methods we use greatly affect their results. This also affects their critical thinking, which is the highest level of knowledge, creating different ideas for solving the task, as well as becoming more creative during their work. Regarding this issue, we have interviewed the two students in question. First, the students discussed among themselves the compilation and solution of the test exercises as well as the way of work they do in the subject of mathematics. The students had the opportunity to express themselves freely during the interview conducted together.

The interview conducted with the student:

For me, the test exercises were very clear and easy, also very attractive, which I finished very quickly and without stress. During the conversation with the friend who had taken the same test, I saw that the afternoon work helped me a lot because he always tried to make the lessons easier for us with illustrations and more activities during the lesson.

In the two classes, we tried to get the opinion of each student regarding the application of practical methods, but especially we selected two students from the two classes with excellent success to complete the test and then conduct the interview with each other. During the answers, we noticed that the student who had not practiced the method had problems completing the test since in that class the tasks were solved only mechanically, while the other student did not encounter any difficulties while completing the test (**Figure 13**).

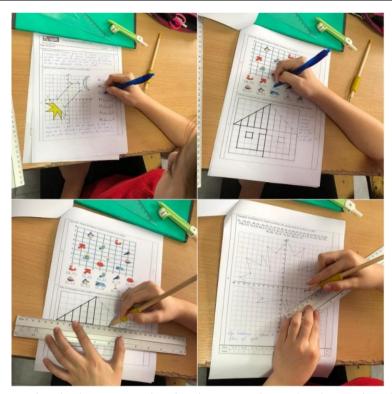


Figure 13. Evidence of the test done by the interviewed student (Source: Evidence of student work on the graph of the function)

## The interview conducted with the student:

I just saw the test, I felt worried, and I said to myself that this is how we learned, and I thought that they are very difficult because I did not learn them in this way, but in a mechanical way. I did not think that presenting the points in the coordinate system creates the feeling as if you are playing a game, I learned this from my friend who showed me the progress of the way of solving the test tasks. After the conversation, I changed the opinion I had when I started working on the test and I noticed that the way of working, and the application of different methods affects us a lot.

The interview conducted with two students gave us as teachers the courage to continue researching new work methods and techniques to create students' comfort and desire to learn the subject of mathematics because the result will never be missing.

## **CONCLUSIONS AND RECOMMENDATIONS**

So far during our work as teachers, based on the test results, the interview with the student, the questionnaire with the student, and the close correlation of the variables in the questionnaire, we have noticed that the more attractive we make the class to students and the more accessible the unit through illustrations, demonstrations and the result will not be missing. As we harness the capabilities of GeoGebra to model, graph, and manipulate functions, we not only enhance our mathematical skills but also cultivate a deeper understanding of the fundamental principles that govern our real dynamic world. It is much more enjoyable for a student if he learns the graphical representation of the function by waiting with great enthusiasm for what figure will be created for us in the end by joining the dots (pairs). On the contrary, if we only told them to graphically present this function, they would find it more boring than to use the method mentioned above.

Throughout this research, as teachers, we recommend everyone to use the internet to unfold the various ideas we can find and practice with their students.

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**Ethical statement:** The authors stated that, according to applicable institutional regulations, the study does not require approval from an ethics committee since it is educational research based on standardized tests. Informed consent was obtained from the participants before the research was conducted; and they were informed about the research, anonymity and their rights about withdrawing from the study without adverse repercussions. All students participate in the study with desire, will and full awareness.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

## REFERENCES

- Aliu, A., Rexhepi, S., & Iseni, E. (2021). Analysis and comparison of commitment, homework, extra hours, preliminary grades, and testing of students in mathematics using linear regression model. *Mathematics Teaching Research Journal*, *13*(3), 21-52.
- Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic Issues for Learning.
- Elia, I., Panaoura, A., Eracleous, A., & Gagatsis, A. (2007). Relations between secondary pupils' conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education, 5*, 533-556. https://doi.org/10.1007/s10763-006-9054-7
- Janvier, C. E. (1987). Problems of representation in the teaching and learning of mathematics. Lawrence Erlbaum Associates.
- Kamberi, S., Latifi, I., Rexhepi, S., & Iseni, E. (2022). The influence of practical illustrations on the meaning and operation of fractions in sixth grade students, Kosovo-curricula. *International Electronic Journal of Mathematics Education*, 17(4), Article em0717. https://doi.org/10.29333/iejme/12517
- Kieran, C. (1992). The learning and teaching of school algebra. In A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 390–419). Macmillan Publishing Co.
- Mollakuqe, V., Rexhepi, S., & Iseni, E. (2021). Incorporating Geogebra into teaching circle properties at high school level and its comparison with the classical method of teaching. *International Electronic Journal of Mathematics Education*, *16*(1), Article em0616. https://doi.org/10.29333/iejme/9283
- OECD (Organization for Economic Cooperation and Development). (2017). PISA 2015 assessment and analytical framework: Science, reading, mathematic, financial literacy and collaborative problem solving (revised ed.). OECD Publishing.
- Panaoura, R., Paraskevi, M. C., & Philippou, A. (2015). Teaching the concept of function: Definition and problem solving. *Ninth Congress of the European Society for Research in Mathematics Education* (pp. 440-445). Charles University in Prague.
- Rexhepi, S., Iseni, E., & Kera, S. (2021). Introduction to mathematical statistics with exercises. Gostivar.
- Sajka, M. (2003). A secondary school students' understanding of the concept of function a case study. *Educational Studies in Mathematics*, 53, 229-254. https://doi.org/10.1023/A:1026033415747
- Shehu, I., Gjergji, R., & Kadriu, M. (2019). Matematika per klasen e 10 te arsimit te mesem te larte [Mathematics for the 10th Grade of Upper Secondary Education]. Prishtine.
- Steele, M. D., Hillen, A. F., & Smith, M. S. (2013). Developing mathematical knowledge for teaching in a methods course: The case of function. *Journal of Mathematics Teacher Education*, *16*, 451-482. https://doi.org/10.1007/s10857-013-9243-6
- Tuda, S., & Rexhepi, S. (2023). Exploring exponential functions using Geogebra. *Brillo Journal*, 3(1), 43-58. https://doi.org/10.56773/bj.v3i1.45
- Tuda, S., & Rexhepi, S. (2024). Geogebra impact in avoiding common mistakes students make in handling exponential functions. *Mathematics & Informatics*, 67(4). https://doi.org/10.53656/math2024-4-7-geo
- Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education*, 20, 356-366. https://doi.org/10.2307/749441
- Zejnullahu, R., Hamiti, E., Vula, E., & Bilalli, S. (2004). Mathematics 7. Prishtine.