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MODESTUM

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Using the graduated sequence instruction to combat errors in pre-calculus

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ABSTRACT

The position of developing countries on international mathematics assessments lends to a growing concern among stakeholders. The factors are multifaceted. This explanatory sequential mixed research design used the graduated sequence instruction (GSI) model to help ameliorate the situation. The researchers sampled 100 students: 50 (experimental group) received GSI and the other 50 (control group) received traditional whole-class instruction. Participants were assessed before and after the treatments. Analysis of covariance was performed to determine the effects between the groups while controlling for pre-test, sex, and age. The results indicate significant differences between the groups. However, sex and age showed no significant differences. The qualitative texts confirmed conceptual errors in traditional instruction. It was recommended that the GSI model be employed in teaching and learning basic concepts in pre-calculus.

Keywords: combat errors, experiential learning, graduated sequence instruction, pre-calculus

INTRODUCTION

Calculus seeks to solve problems in tangent and area/volume (Berggren, 2021). Regrettably, senior high school students continue to perform abysmally (Alam, 2020; Hurdle et al., 2022; Karikari et al., 2020; National Teaching Council [NTC], 2023) due to conception, applications, and formulas (Aduko & Armah, 2022; Baidoo & Ali, 2024; Domondon & Rin, 2022; Owusu-Darko et al., 2022). West Africa Examination Council (2020) found that traditional instruction breeds rules and procedures without conceptual understanding (Ali, 2021, 2023; Owusu-Darko et al., 2023). One school (Agbofa, 2023; Bouck et al., 2019; Butakor & Dziwornu, 2018; Varaidzai & Makondo, 2020) attributed the canker to schools, and another (Abaidoo, 2018; Arhin & Hokor, 2021; Owusu-Darko, Apoenchir & Mensah, 2022) to the environment. This study believes that the graduated sequence instruction (GSI) could be the stem tide.

GSI, also known as concrete-representational-abstract, visual-representation-multi-sensory approach, peer-assisted reflection, or concreteness fading (Ali, 2024; Bustos Tiemann & Ramos Rodriguez, 2022) sequentially and gradually promotes and propels one's conceptual understanding, procedural accuracy, and multi-sensory within local context (Ali, 2023; Flores & Hinton, 2022). Moving from concrete through visual representations aids deeper understanding (Litteck et al., 2024), relates to real-world representations (e.g., graphs, tables, and diagrams) (Baidoo & Ali, 2023; Herrera et al., 2024; Hong & Lee, 2022; Parr et al., 2024), and enhances conceptual, visualization, problem-solving, and engagement skills (Lemonidis et al., 2020; Spektar, 2023).

Calculus as a Function

A 'function' is an ordered pair such as that $x \rightarrow f(a, b)$ or an input-output such that $x \rightarrow f(x)$. This rule assigns every element in the first set to a unique element in the second set (García-García & Dolores-Flores, 2020; Nagle et al., 2019). However, the word 'function' could ambiguously mean a social gathering, and 'input-output' could mean the value is the same (Hatisaru, 2023).

Calculus as a Limit

Let $f(x)$ be a function and L as real numbers. Then as $'x'$ approaches 'a', as $f(x)$ approaches L. The notation $\lim_{x \rightarrow a} f(x) = L$ means as x tends to 'a' $(x \rightarrow a)$, 'x' gets closer and closer to $f(x)$ (Radmehr & Drake, 2020). However, students think 'never reaches' means the function cannot be evaluated (Sidelil, 2019).

Calculus as a Derivative

The derivative is $\frac{dy}{dx}$ and the integral is $\int f(x)dx$. Here, students cannot establish the relationship between the differential and integral (Borji et al., 2018). Consequently, they cannot conceive $\frac{dy}{dx}$ as a 'single fraction' (Nieto et al., 2022; Radmehr & Turgu, 2024; Sie & Agyei, 2023).

Calculus as Fundamental Theorem

The fundamental theorem says that derivatives and antiderivatives are inverse processes. That is if f is continuous on [a, b] and F is an antiderivative of f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$, and if $\int_a^x f(t)dt$ is an antiderivative of f, then $\frac{d}{dx}[\int_a^x f(t)dt] = f(x)$. However, the difficulties are functional notations and lack of knowledge in 'f' and 'F' (Radmehr & Turgu, 2024).

Therefore, this study seeks to address the following research questions:

1. What errors do students commit?
2. How does the GSI model combat the errors?

METHODOLOGY

Research Design

Domondon et al. (2022) and Spekta (2023) used quantitative-qualitative methods. Brannen (2018) used mixed-method simultaneously and sequentially. Gesser-Edelsburg et al. (2020 used a quantitative method dominated by a qualitative. This study adopts a sequential explanatory design from Creswell and Creswell (2018) to analyze both quantitative and qualitative data (Douglas et al., 2020).

Population and Sample

The population was 318 students. About 100 participated in the quantitative phase through simple random sampling techniques (Nga et al., 2023). They comprised girls and boys across general science, general arts, home economics, business, and technical/vocational programs. They also included form 1, form 2, and form 3 students who hailed from rural, peri-urban, and urban, with diverse social, economic, and cultural backgrounds. These ensured the representativeness and generalizability of the findings (Nagle et al., 2019). Ten participants were used in the qualitative phase through purposive sampling techniques (Berggren, 2021).

Data Collection Procedure

The participants in the experimental group were designated SE_1, SE_2, SE_3, …, and SE_{50}, and control as SC_1, SC_2, SC_3, …, and SC_{50}. Even though both groups were pre-tested, only the experimental group was administered with the GSI model (Listiawati & Juniati, 2021). In the qualitative phase, the text scripts were scanned and subjected to content analysis (Schoonenboom & Johnson, 2017).

Sustainability and Impact

The graduate school stipulates that a graduate research student starts the thesis after one year of coursework. So, we started the research in October 2021. By October 2023, the data collection processes had been completed. Therefore, the two-year duration was enough to create sustainability (CKT-UTAS, 2021). The researchers also maintained methodological consistency by analyzing both quantitative and qualitative data to reveal errors and guarantee research sustainability. The GSI theory is time-tested and has existed for many 25 years. The data processing, and statistical significance in the quantitative, and data categorization and excerpts in the qualitative suffice more sustainability (Torres et al., 2024).

Research impact was measured by the transformation of 55 academic literature. We had 30 on calculus, 20 on GSI, and 5 on methodology. The development of the interventions treatment activities, and social interactions foster higher-quality impact (Torres et al., 2024).

Data Collection Instruments

The four sections of the instruments were demographic characteristics, causes of poor performance, the GSI model, and research sustainability. The test items were closed-ended and open-ended (Da, 2023). Items 1, 2, 3, and 16 were adapted from Sebsibe and Feza (2020), and items 6, 7, 8, 9, 10, 11, 14, 15, and 17 from West Africa Examination Council (2020). We used digital recorders, transcription software, and visual analysis software to collect qualitative data. The codes and themes were generated to observe patterns and relationships (Principe, 2022). We reduced respondent biases by avoiding sensitive or controversial questions. We reduced researcher biases with multiple coders, external reviews, triangulations, and participants' evaluations (Principe, 2022).

Potential Scalability and Adaptability

Scalability is the ability of an instrument to effortlessly transition back and forth from micro to macro (Flyvbjerg, 2020; Milat et al., 2020). The researchers adopted the Vaughan-Lee et al. (2018) framework involving success, adaptability, sustainability, and high effectiveness. We expanded the scope and content of the study (Bulthuis et al., 2022; Vaughan-Lee et al., 2018). The GSI model

Table 1. Key errors

Error		GSI function
Carelessness	Dropping the constant	Corrective feedback, collaboration, and re-enforcement
	Dropping negative signs	
	Writing questions wrongly	
	Sloppy handwriting	
	Not stating units	
Procedural	Incorrect computation	Corrective feedback and practice
	Overgeneralization writing $\infty^0 = 1 \; and \; \frac{\infty}{\infty} = 1$	
Conceptual	Taking infinity as numbers	Modelling, formulation, and practice
	Writing $\frac{0}{0} = 0 \; and \; \frac{\infty}{0} = \infty$	
	Confusing functional values of limits	
	Misunderstanding limits as unreachable, and infinite	
	Confusing notation $f^{-\prime}(x)$ as $f'(x)$	
	Linguistic issues	

was made adaptable to different contexts, content, and target groups (Milat et al., 2020). We also collected data for two years to sustain horizontal (quantitative) and vertical (qualitative texts) studies. The scalability was mostly observed in the piloting, text scripts, and GSI intervention model (Palmie et al., 2023).

Data Analysis Procedures

The quantitative data was analyzed t-test and analysis of covariance (Hurdle et al., 2022). The qualitative phase was analyzed by codes and plausible themes. Excerpts were scanned and pasted for comparative analysis.

Reliability and Validity

We checked the reliability coefficient with the Kuder-Richardson formula 20 (Baidoo & Ali, 2023). We also satisfied content, construct, and criterion-related validities, and internal and external threats to validity (Moloto & Machaba, 2021; Sie & Agyei, 2023). The researchers randomly assigned participants to both experimental and control groups to ensure comparability and eliminate biases and triangulated them with the traditional and GSI models (Ali, 2024).

Ethical Considerations

Approval was obtained from the Ethical Committee of the Univerisity of Education, Winneba. The following guidelines were adopted by CKT-UTAS (2021):

1. The purpose of the study and the participants' rights
2. Participation was strictly voluntary by signing the consent form.
3. Participants were fully informed of their right to withdraw from the study at any time.
4. Participants were fully informed that they would take assessments.
5. The schools were written to for permission.
6. The privacy and confidentiality of participants were assured

Challenges During Implementation

The major hiccups in the implementation emanated in the GSI model during the training of moderators, matching, and triangulation.

RESULTS AND DISCUSSION

In the first part, we present the quantitative results. In the second section, we present the text analysis.

Research Question One: Students' Errors

In **Table 1**, the students encountered real-life settings, modelling, problem-solving, communication, and collaborative skills. The GSI model was responsive, conducive, and viable.

Research Question Two: Minimizing Errors

This research question examined the cause-effect relationships with a hypothesis:

H_0: There is no significant difference before the GSI model.

The analysis satisfied the following assumptions:

1. Linearity of model
2. Linearity between covariates and outcomes
3. No interaction between factors and covariates

Table 2. Descriptive statistics (N = 50)

Group	Pre-test		Post-test (unadjusted)		Post-test (adjusted)	
	M	SE	M	SE	M	SE
Experimental	51.64	0.87	80.10	1.27	79.91	0.83
Control	51.16	0.86	58.90	1.03	59.09	0.83

Table 3. Independent samples t-test for post-test

Levene's test for equality of variances			t	df	Sig. (2-tailed)	Mean difference	SE difference	95% CI of the fifference	
	F	Sig.						Lower	Upper
Equal variances assumed	1.345	0.249	12.993	98.000	0.000	21.20000	1.63170	17.96194	24.43806
Equal variances are not assumed				94.088				17.96026	24.43974

Table 4. Analysis of covariance for covariates

Source	Type III sum of squares	df	Mean square	F	Significance	Partial eta squared
Pre-test	2,560.086	1	2,560.086	75.261	0.000	0.442
Sex	207.169	1	207.169	6.090	0.015	0.060
Age	25.143	1	25.143	0.739	0.392	0.008
Group	10,812.443	1	10,812.443	317.864	0.000	0.770
Error	3231.509	95	34.016			

Note. [a]R squared = .818 (adjusted R squared = .810)

Table 5. Texts on limits

Items		A		B		C		D		Non-respondents	
		N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
Experimental	1.1	2	4	16	32	4	8	28*	56	0	0
	1.2	0	0	1	2	2	4	47*	94	0	0
	1.3	0	0	46*	92	0	0	4	8	0	0
	1.4	4	8	1	2	41*	82	3	6	0	0
	1.5	2	4	1	2	3	6	43*	86	1	2
Control	1.1	3	6	38	76	4	8	2*	4	3	6
	1.2	3	6	2	4	1	2	43*	86	1	2
	1.3	15	30	14*	28	3	6	18	36	0	0
	1.4	11	22	6	12	24*	48	2	4	7	14
	1.5	7	14	1	2	27	54	15*	30	0	0

Note. P: Percentage

4. Equal variances
5. Normality of residuals

In **Table 2**, the experimental (mean [M] = 51.64, standard error [SE] = 0.87) and control (M = 51.16, SE = 0.86) groups are nearly the same. However, the unadjusted experimental group (M = 80.10, SE = 1.27) was greater than the control group (M = 59.90, SE = 1.03).

Tests of Hypotheses

H_{01}: There is no statistical significance between the group performance.

This t-test compared the performance before the effect of the pre-test, age, and sex were statistically removed.

In **Table 3**, the t-test (t [98] = 12.993, p = 0.000) indicates that there are group mean differences. The null hypothesis was therefore rejected since the experimental group outperformed the control group.

H_{02}: There is no statistical significance between the group performance.

The ANCOVA was used to remove the statistical effects of the pre-test, age, and sex.

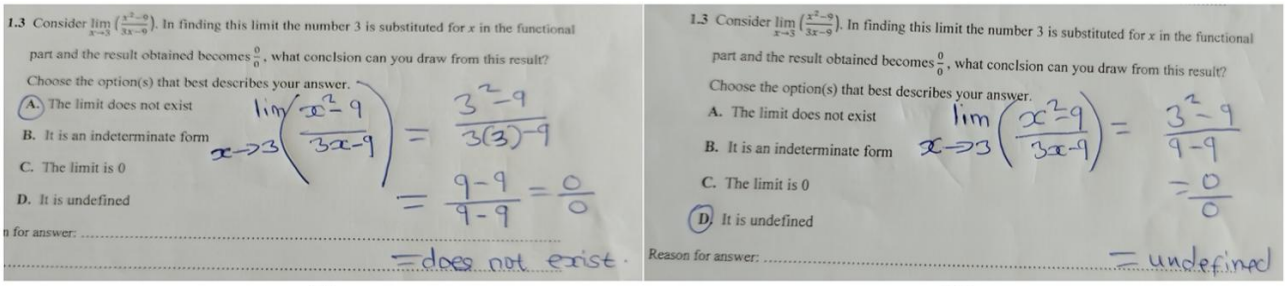
In **Table 4**, the results (F [1, 95] = 317.864, p = 0.000) indicate there is a significant difference. We therefore reject the null hypothesis as the students in the GSI model outperformed their peers. However, the effect of sex (F [1, 98] = 6.090, p = 0.015) and pre-test (F [1, 98] = 75.261, p = 0.000) were significant.

Qualitative Text Analyses

In **Table 5**, item 1.1 scored 28 (56%) for the experimental group in limit value and the function whereas only 2 (4%) for the control group. Item 1.2, 46 (94%) of the experimental group and 43 (86%) in the control group solved the at $x = 3$. Item 1.3 was well understood by the experimental group 46 (92%) as compared to only (8%) in the control group in the $\frac{0}{0}$ is undefined.

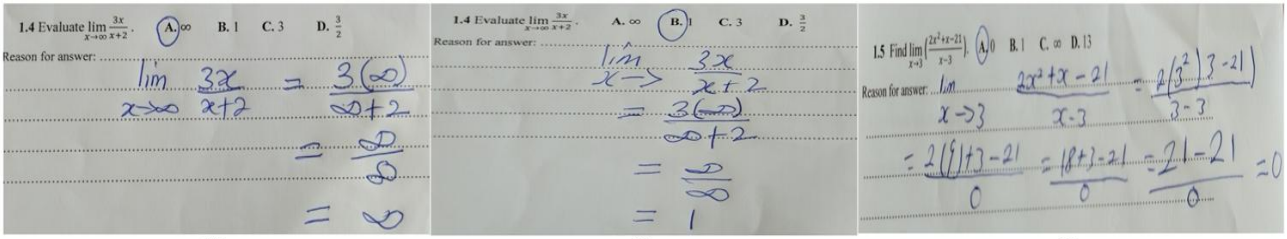
Figure 1 indicates that items 1.4 and 1.5 record 41 (82%) and 43 (86%) correct responses for the experimental group and 14 (28%) and 15 (30%) in the control group. Two major difficulties were taking infinity as a number and generalizing $\frac{\infty}{\infty} = 1$ or $\frac{\infty}{\infty} = \infty$.

In **Figure 2**, the errors are both procedural and conceptual difficulties.



CS5 CS18

Figure 1. Two excerpts in the control group-1 (Source: Field study, Control Participants 5 & 18)



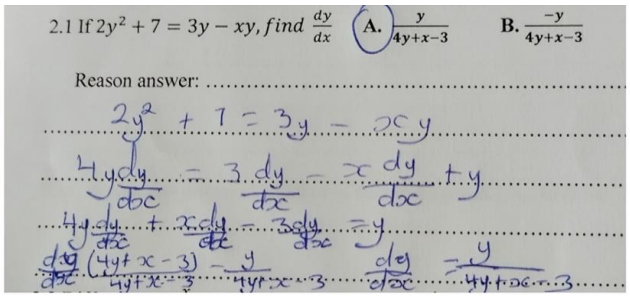
CS23 CS27 CS48

Figure 2. Three excerpts in the control group (Source: Field study, Control Participants 23, 27 & 48)

Table 6. Texts on derivatives

Items		A		B		C		D		Non-respondents	
		N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
Experimental	2.1	1	2	48*	96	0	0	1	2	0	0
	2.2	42*	84	0	0	7	14	1	2	0	0
	2.3	1	2	3	6	46*	92	0	0	0	0
	2.4	5	10	39*	78	2	4	2	4	2	4
	2.5	4	8	2	4	3	6	37*	74	4	8
Control	2.1	20	40	22*	44	7	14	2	4	1	2
	2.2	24*	48	1	2	20	40	2	4	3	6
	2.3	2	4	5	10	40*	80	1	2	2	4
	2.4	5	10	25*	50	6	12	4	8	10	20
	2.5	3	6	8	16	9	18	23*	46	7	14

Note. P: Percentage



CS17

Figure 3. One excerpt in the control group (Source: Field study, Control Participant 17)

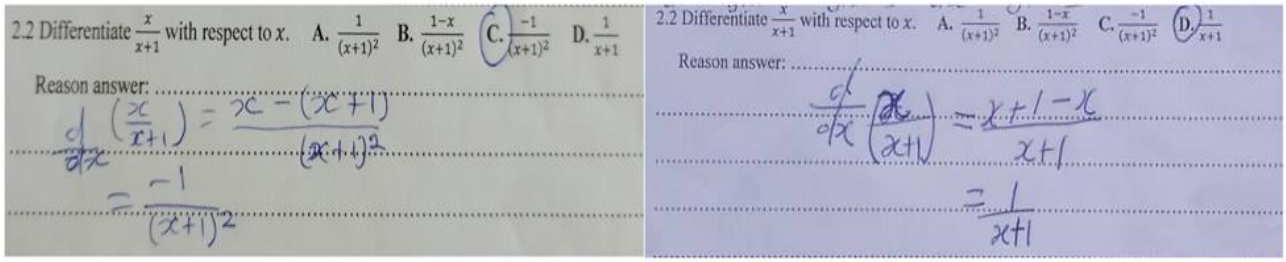
In **Table 6**, items 2.1-2.5 showed that 96% of the experimental group correctly answered item 2.1, while 2 (4%) missed it. For the control group, 22 (44%) correctly answered item 2.1, and 28 (56%) missed it.

In **Figure 3**, 84% of the experimental group correctly answered it whereas 16% missed it. In the control group, 52% missed it.

The difficulties observed with the control group's notational order, writing $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{u\frac{dv}{dx}-v\frac{du}{dx}}{v^2}$ instead of $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$, And carelessness with negative signs.

In **Figure 4**, 30% of the control group failed to recognize the function of a product. They simply differentiated it as though it was a function of a function, and this led to incorrect decisions.

In **Table 7**, 92% of the experimental group answered item 16A correctly, and only 8% got it wrong. In the control group, 54% correctly answered the item, 34% had it wrong and the remaining 12% left the item unanswered. In items 16B, 16C, and 16D; 86%, 78%, and 54% in the experimental group answered them correctly. Careful examinations of their work show that the difficulties were taking 'function value as 'limit value', incorrect computation, and sloppy handwriting.



SC12 SC15

Figure 4. Two excerpts in the control group-2 (Source: Field study, Control Participants 12 & 15)

Table 7. Texts on item 16

Item	Experimental group						Control group					
	Correct		Incorrect		Non-response		Correct		Incorrect		Non-response	
	N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
16A	46	92	4	8	0	0	27	54	17	34	6	12
16B	43	86	7	14	0	0	23	46	20	40	7	14
16C	39	78	9	18	2	4	21	42	18	36	11	22
16D	28	56	16	32	6	12	19	38	16	32	15	30

Note. P: Percentage

SC6

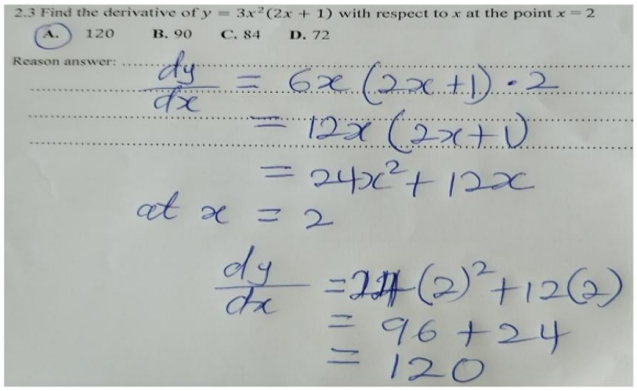
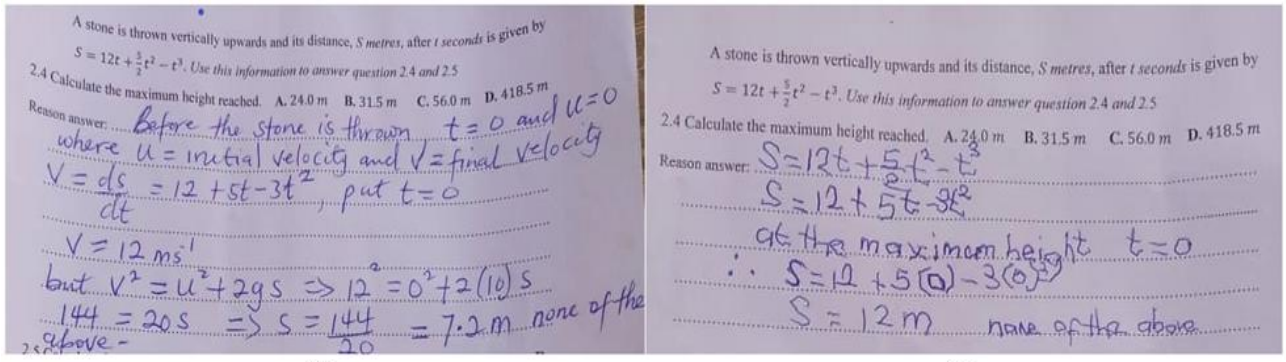


Figure 5. One excerpt on item 'A' (Source: Field study, Control Participant 4)



SC11 SC38

Figure 6. Two excerpts on item 2.4 (Source: Field study, Control Participants 11 & 21)

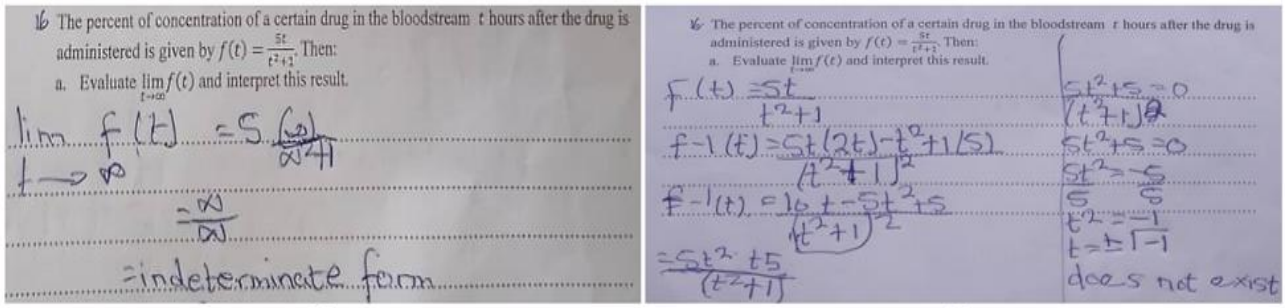
In **Figure 5**, 78% in the experimental group provided correct responses, and only 50% in the control group responded correctly. The item highlighted conceptual and procedural errors as the main difficulties.

In **Figure 6**, the difficulties were associated with derivatives in the experimental group.

In **Figure 7**, the difficulties were recorded;

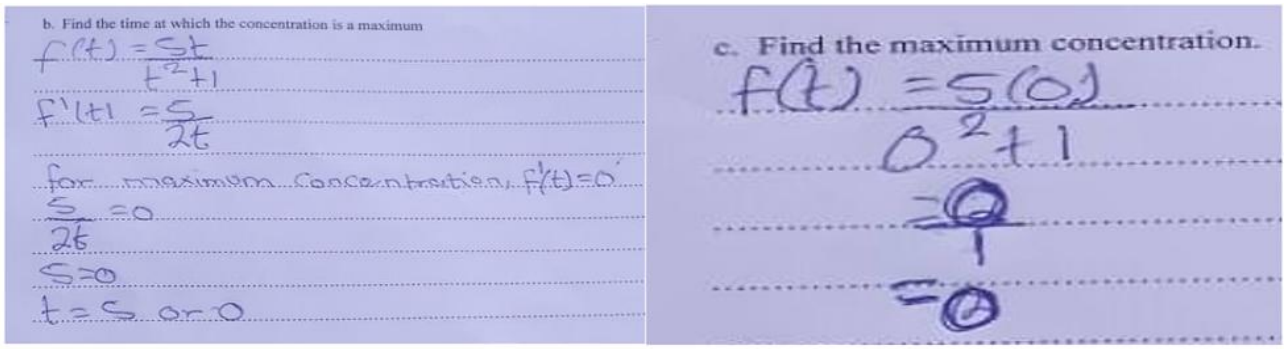
1. Confusion with notations like $f^{-\prime}(x)$ instead of $f'(x)$.
2. Failure to realize that at the maximum concentration, $f'(x) = 0$.
3. Difficulties factorizing quadratic equation.

Figure 8 shows two excerpts.



SC27 SC39

Figure 7. Two excerpts on item 16 (Source: Field study, Control Participants 21 & 39)



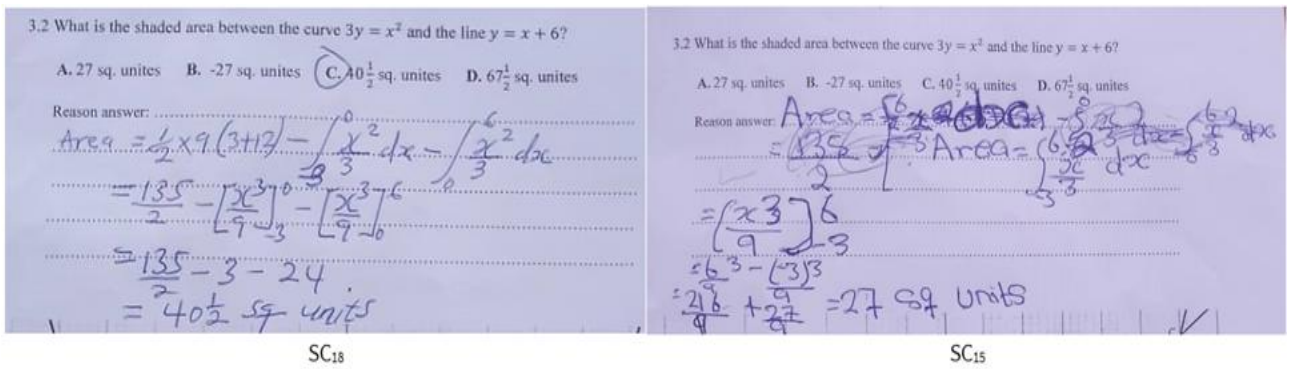
SC19 SC50

Figure 8. Two excerpts-1 (Source: Field study, Control Participants 29 & 50)

Table 8. Texts on integrals

Items		A		B		C		D		Non-respondents	
		N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
Experimental	3.1	40*	80	5	10	3	6	2	4	0	0
	3.2	3	6	2	4	43*	86	2	4	0	0
	3.3	1	2	1	2	2	4	46*	92	0	0
Control	3.1	25*	50	4	8	5	10	9	18	7	14
	3.2	5	10	6	12	20*	40	4	8	15	30
	3.3	1	2	4	8	5	10	24*	48	16	32

Note. P: Percentage



SC18 SC15

Figure 9. Two excerpts-2 (Source: Field study, Control Participants 13 & 15)

Texts on Integrals

Items 3.1-3.3 were closed-ended to evaluate knowledge of integrals.

In **Table 8**, 40% of the control group were correct, 30% left the item unanswered and 30% got the item wrong. Options A, B, and D were distractors arrived at from students' failures to sketch the function and compute the area of the curve.

On item 3.3, 92% of the experimental group answered the item correctly, while only 8% had it wrong. However, students in the control group (52%) answered it wrongly.

Figure 9 shows some excerpts.

Table 9. Responses to item 17

Item	Experimental group						Control group					
	Correct		Incorrect		Non-response		Correct		Incorrect		Non-response	
	N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
17A	42	84	8	16	0	0	20	40	27	54	3	6
17B	41	82	9	18	0	0	19	38	20	40	11	22
17C	48	96	2	4	0	0	28	56	16	32	15	30

Note. P: Percentage

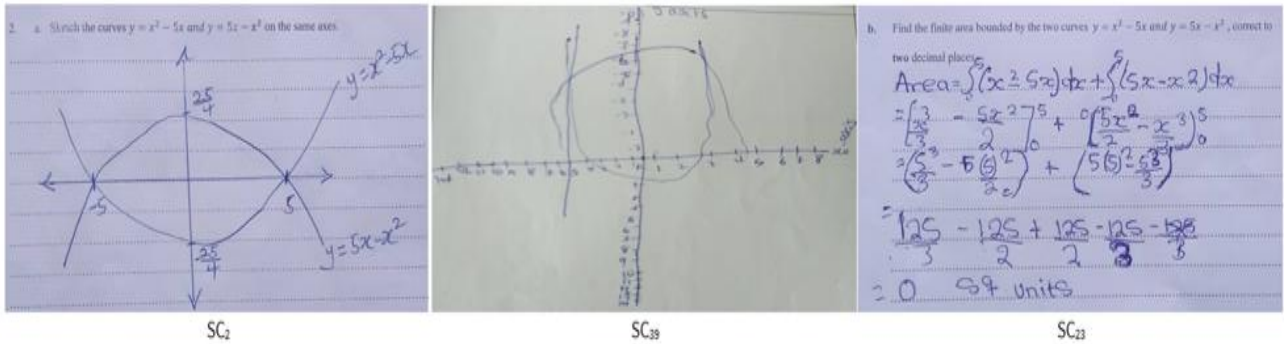


Figure 10. Excerpts of students' responses on graphing (Source: Field study, Control Participants 1, 20 & 29)

Table 10. Texts on FTC

Items		A		B		C		D		Non-respondents	
		N	P (%)	N	P (%)	N	P (%)	N	P (%)	N	P (%)
Experimental	4.1	2	4	1	2	3	6	44*	88	0	0
	4.2	1	2	3	6	43*	86	2	4	1	2
Control	4.1	7	14	9	18	9	18	24*	48	1	2
	4.2	12	24	2	4	25*	50	10	20	1	2

Note. P: Percentage

In **Table 9**, 84% of the experimental group sketched 17A, while only 16% had an incorrect curve. In the control group, 40% sketched it, 54% had incorrect sketches and 6% left it unanswered. In 17B, 82% of the experimental group answered well, while 18% had difficulties. For the control group, only 38% had it correct while 40% had it wrong, and as high as 22% left it unanswered. For item 17C, 96% of the experimental group had correct answers and 4% were incorrect. In the control group, 56% had it correct 32% wrong, and 30% left it unanswered. The difficulties were inappropriate scale, x and yy-intercepts wrong computations, and units.

Figure 10 shows excerpts of students' responses on graphing.

Students' Text Analysis on FTC

In **Table 10**, the experimental group achieved 88% proficiency whereas the control group demonstrated 48%.

DISCUSSION

The findings indicate that the traditional approach was severely weak. The poor performance was corroborated by Butakor and Dziwornu's (2018) findings of irregular training models in calculus (West Africa Examination Council, 2020). Alam (2020) suggests a blend of interconnected strategies. However, the GSI model brought modelling, practicing, and re-enforcement components to allow students to visualize, think-pair-share, collaborate, and communicate (Ali, 2023; Owusu-Darko et al., 2022).

Undoubtedly, calculus is the nucleus for pursuing STEM programs (Da, 2023). Comparing the groups, the experimental outperformed the control (Tettey et al., 2018) due to the GSI model's enhanced conceptual understanding, visualization, problem-solving, and engagement skills (Bouck et al., 2019) and guided them to model calculus (Hinton & Flores, 2019). Findings recommend remedial lessons, remediations, multiple representations, and information and communication technology tools to augment the model (Lemonidis et al., 2020; Spektar, 2023).

CONCLUSION

The experimental group outperformed their peers in the control group. The GSI model yielded the high significant differences.

The qualitative texts portrayed ambiguity and procedural and conceptual errors. This was accomplished by the GSI model. It was recommended that the GSI model be employed to detect and reduce errors before teaching and learning basic concepts.

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